إهداء

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Abstract

This thesis aims to design and analyses passiveand active filters, the research included identify of main concepts which required to understanding filters as a two port structure, such as the time domain, frequency domain, and s domain, beside to some filters applications.

Filters types have been illustrated, the classes which according on their characteristics and response shape have been studied widly.

Tables method of design passive LPF prototype filters also have been illustrated, beside to transfer rules from LPF to HPF and BPF.

Two main methods of design active LPF prototype filters also have been illustrated, first one related with second order active filters which have several shapes, one of them called sallenkey (SK), this type has been studied, beside to transfer rules from LPF to HPF. The other methods related with filters which have order higher than two, in this worke frequency dependent negative resistance simulation method has been studied.

2^{th} order sallen key (SK) LPF has been designed, components values have been calculated either, and the circuit has been simulated and implemented.

Chapter 1

Introduction

Filter structure

Lumped LCR – ladder filters comprising resistors, capacitors and inductors terminated at both ends with resistors serve to realize transfer functions with poles in the left half s-plane and zeros on the $j\omega$axis thereby allowing the synthesis of standard approximation functions such as Butterworth, Chebyshev and elliptic filters. The most important feature of conventional LCR ladder networks is that they can be designed to have very low sensitivity in the passband.

Unfortunately, these filters are not compatible with integrated circuit technology which dominates most of today's systems. This is mainly because indouctors are not suitable for miniaturization through integration. On the other hand, capacitors and resistors can be integrated using hybrid circuit technology. It is also an advantage to replace inductors in the prototype network even if they are not to be integrated, since practical inductors in the prototype networks even if they are not to be integrated, since practical inductors are bulky, comparatively heavy and expensive. Furthermore, this inductor is rather non-ideal device having significant parasitic resistance, capacitance and mutual inductance.

The disadvantages of the inductor were sufficiently great to have caused the field of active filters to develop, using resistors, capacitors and active devices. Active elements, typified by operational amplifier, are now readily and economically available in integrated form.

Hence, the resulting active filter can be constructed at low cost with improvement in electrical and mechanical performance due, essentially, to the absence of the inductor.

Nowadays, active filters present one of the most practical and economically feasible methods of designing inductorless filters.

However these filters are limited gain bandwidth product of practical operational amplifiers. Despite this, RC-active are widely used in many areas including communication and instrumentation systems as they are found to be superior to purely passive filters particularly at low frequencies. At high frequencies the performance of active filters deteriorated from what it expected, although compensation of the operational amplifier degeneracies in active filters can provide a satisfactory improvement in the realized frequency responses.

A very successful design technique for RC-active filters makes use of the concept of component simulation which may be conveniently grouped as follows;

- direct inductance simulation.
- frequency dependent negative resistance simulation.

In direct inductance simulation, an inductorless filter is obtained by first designing the prototype passive LCR ladder filter and then replacing each inductance in the network by an equivalent one-port network of the required values such as a gayrator loaded of the capacitor or a positive impedance converter.

Using the concept of the frequency dependent negative resistance, a passive LCR-ladder filter is transformed in to an equivalent two-port which does not contain inductors. This approach, to be studied here, is typified by the Bruton complex impedance transformation, in which all Circuit impedance are scaled by the factor 1/S.

1.2 Filtering concepts

The word “filter” is in common use, such as an oil filter used in an automobile.

Also used in an automobile is an air filter and a fuel filter. An air filter is alsoused in home heating / air conditioning systems. A lint filter is used in clothesdryer. Photographers frequently make use of a lens filter. In all of these applicationsthe filter is a device that removes something: small metal particles, dust, lint, etc.

The photographic filter suppresses a certain band of wavelengths, or is designed topass light of a particular polarity, etc.

Electric filters may be thought of in a similar way. An electric analog filteris typically designed to pass certain things and attenuate if not completely block otherthings. Since an analog filter is typically time-invariant, what it passes or blocks isnot time-dependent per se. Rather, similar to the photographic filter, it is typicallydesigned to pass certain wavelengths, or frequencies, and attenuate or block others.

Therefore, many of the concepts, and specifications, of filters are defined orexplained in the frequency domain. Just what a given filter accomplishes is muchmore readily comprehended in the frequency domain than in the time domain.

An electric analog filteris typically designed to pass certain things and attenuate if not completely block otherthings. Since an analog filter is typically time-invariant, what it passes or blocks isnot time-dependent, similar to the photographic filter, it is typicallydesigned to pass certain wavelengths, or frequencies, and attenuate or block others.

So in general we say that **filters are circuits which allow a specific range of frequencies to be passed (or rejected) as they are transmitted from an AC source to a load.** Schematically the system is shown on Figure 2.

Fig1. 1 : filters in the system

Filters are generally linear circuits that can be represented as a two-port network. The filter transfer function is given as follows:

$$T(jw) = V_{out} / V_{in}$$

Chapter 2

Filters specification and definitions

2.1Transfer function

Filters are generally linear circuits that can be represented as a two-port network as shown in figure 2.1:

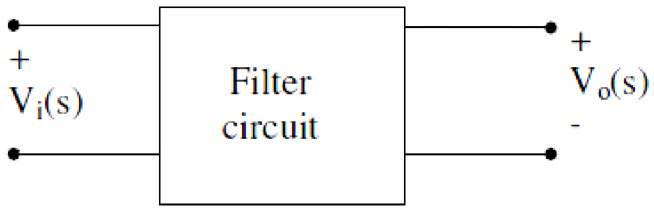

Fig 2.1: ***Filter as two-port network***

The filter transfer function is given as follows:

$$G(j\omega) = G(s) = \frac{V_o\,(s)}{V_i\,(s)} \qquad [1]$$

The transfer function of a circuit is usually expressed on a logarithmic scalein decibels, and since the fundamental quantity of interest is power, a filter ischaracterised by:

$$\mathrm{G}(j\omega) = 20\log_{10}\frac{V_o\,(j\omega)}{V_i\,(j\omega)} \qquad [2]$$

The symbols and Bode diagrams for transfer functions for thesefilters are shown in Figure 2.2, thus, curves of gain vs. frequency are commonly used to illustrate filtercharacteristics,and the most widely-used mathematicaltools are based in the frequency domain.

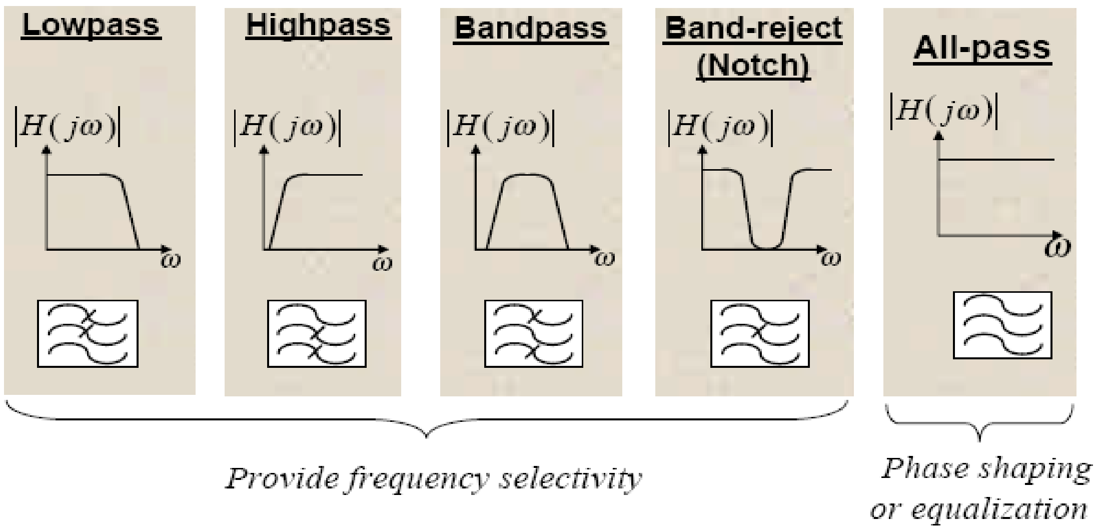

Fig 2.2 : The symbols and Bode diagrams for transfer functions for main filters

2.2 Frequency domain and time domain

When the response of a circuit is described to a sinusoidal input, the description is referred to the frequency domain description. The response is the transfer function and is a function of the frequency of the input signal. The alternative is a time domain description. For instance, how would the system respond to a step function input? Here the response function is a function of time, in particular the time after the application of the step change in the input. They are two different ways of looking at the same system, and knowledge of one response function will allow us to calculate the other, at least in principle.

2.3 Fourier transforms

Frequency domain analysis and Fourier transforms is a cornerstone of signaland system analysis.Using the Fourier Transform a time domain signal is transformed to the frequency domain, where it is equivalent to an Amplitude Spectrum and a Phase Spectrum.

Fourier series and Fourier transform are ways to find spectrumsfor periodic and aperiodic signals.For any function f(t) with period 2π(f(t) = f(2π+t)), we can describe the f(t) in terms of an infinite sum of sines and cosines.Alsof(t) can be described by the Fourier series as:

$$x(t) \overset{F}{\leftrightarrow} X(j\omega)$$ [3]

As a result The Fourier transform, X(jw), represents the frequency content of x(t).

2.4The S-Plane

Filtershave a frequency dependent response because the impedance of a capacitor or aninductor changes with frequency. Therefore the complex impedances:

$$Z_L = sL$$ [4]

and

$$Z_c = \frac{1}{\mathrm{sC}}$$ [5]

are used to describe the impedance of an inductor and a capacitor, respectively,
s = σ + jω: complex frequency, whereσ is the Neper frequency in nepers per second (NP/s) and ω is the angularfrequency in radians per sec (rad/s).

By using standard circuit analysis techniques, the transfer equation of the filter can bedeveloped. These techniques include Ohm's law, Kirchoff's voltage and current laws,and superposition, remembering that the impedances are complex. The transfer equationis then:

$$H(s) = \frac{N(s)}{D(s)} = \frac{K(S^M + a_{m-a}\,S^{M-1} + \ldots + a_1\,S + a_0)}{S^N + b_{n-1}\,S^{N-1} + \ldots + b_1\,S + b_0}$$ [6]

Therefore, H(s) is a rational function of s with real coefficients with the degree of m forthe numerator and n for the denominator. The degree of the denominator is the order ofthe filter.

Since filters are defined by their frequency-domain effectson signals, it makes sense that the most useful analyticaland graphical descriptions of filters also fall into the frequencydomain. Thus, curves of gain v_s frequency andphase v_s frequency are commonly used to illustrate filtercharacteristics,and the most widely-used mathematicaltools are based in the frequency domain.

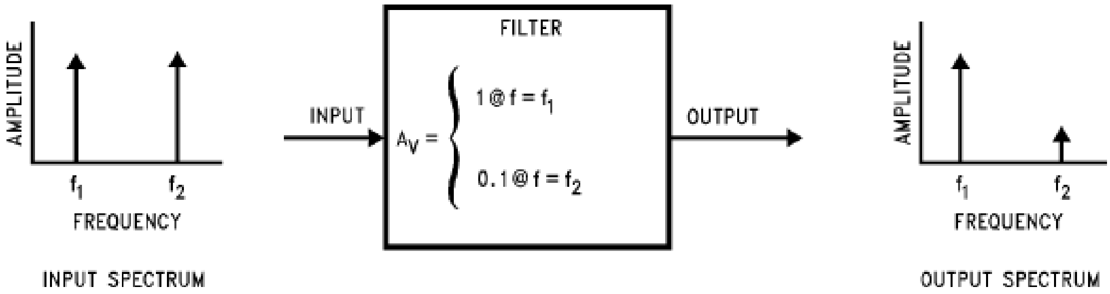

Fig 2.3 : the filter in frequency domain.

The frequency-domain behavior of a filter is described mathematicallyin terms of its transfer function or networkfunction. This is the ratio of the Laplace transforms of itsoutput and input signals. The voltage transfer function H(s)of a filter can therefore be written as:

$$H(s) = \frac{V_{OUT}(s)}{V_{IN}(s)} \qquad [7]$$

where $V_{IN}(s)$ and $V_{OUT}(s)$ are the input and output signalvoltages and s is the complex frequency variable.

The transfer function defines the filter's response to anyarbitrary input signal, but we are most often concerned withits effect on continuous sine waves. Especially important isthe magnitude of the transfer function as a function of frequency,which indicates the effect of the filter on the amplitudesof sinusoidal signals at various frequencies. Knowingthe transfer function magnitude (or gain) at each frequencyallows us to determine how well the filter can distinguishbetween signals at different frequencies. The transfer functionmagnitude versus frequency is called the amplituderesponse or sometimes, especially in audio applications,the frequency response.

2.5Complex Poles and stability

Filter's transfer function can be written in the form:

$$T(s) = \frac{K(s-Z_1)(s-Z_2)\ldots.(s-Z_M)}{(s-P_1)(s-P_2)\ldots..(s-P_N)} \qquad [8]$$

Where the roots of the numerator, Z_0, Z_1, Z_2, . . .Z_n are known aszeros, and the roots of the denominator, P_0, P_1, . . . P_n arecalled poles. Z_i and P_i are in general complex numbers, i.e.,R + jI, where R is the real part, j = $\sqrt{-1}$, and I is theimaginary part. All of the poles and zeros will be either realroots (with no imaginary part) or complex conjugate pairs. Acomplex conjugate pair consists of two roots, each of whichhas a real part and an imaginary part. The imaginary parts ofthe two members of a complex conjugate pair will have oppositesigns and the real parts will be equal.

Solving for the roots of the equation determines the poles (denominator) andzeros (numerator) of the circuit. Each pole will provide a –6 dB/octave or –20 dB/decaderesponse. Each zero will provide a +6 dB/octave or +20 dB/decade response. These rootscan be real or complex. When they are complex, they occur in conjugate pairs.

Theseroots are plotted on the s plane (complex plane) where the horizontal axis is σ (real axis)and the vertical axis is ω (imaginary axis). How these roots are distributed on the s planecan tell us many things about the circuit. In order to have stability, all poles must be inthe left side of the plane. If we have a zero at the origin, that is a zero in the numerator,the filter will have no response at dc (high-pass or band pass).

With only resistors and capacitors, there is only real poles. For complex poles, either an op-amp or an inductor will be needed.

Complex poles allow you to build better low pass filters, high pass filters, band pass filters, and bandreject filters. Graphically, this works as follows.Consider the transfer function:

$$Y = \left(\frac{1}{S + a}\right) X \qquad [9]$$

The frequency response is obtained by letting $s \to j\omega$:

$$Y = \left(\frac{1}{j\omega + a}\right) X \qquad [10]$$

Graphically, the gain is equal to the vector '1' divided by the vector 'jω + a'. The latter term is equal tothe vector from the pole at -a to the origin (a) plus the vector jω.

The pole-zero diagrams can be helpful to filter designers asan aid in visually obtaining some insight into a network'scharacteristics as in figure 2.4 A pole anywhere to the right of the imaginaryaxis indicates instability. If the pole is located on thepositive real axis, the network output will be an increasingexponential function. A positive pole not located on the realaxiswill give an exponentially increasing sinusoidal output.We obviously want to avoid filter designs with poles in theright half-plane.

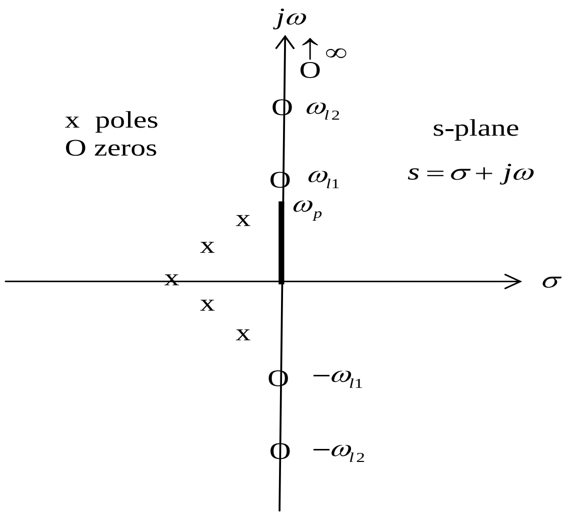

Fig 2.4 : A typical pole-zero pattern for a 5th order low-pass filter

Stable networks will have their poles located on or to theleft of the imaginary axis. Poles on the imaginary axis indicatean undamped sinusoidal output (in other words, a sinewaveoscillator), while poles on the left real axis indicatedamped exponential response, and complex poles in thenegative half-plane indicate damped sinusoidal response.**Also we must not forget; for the filter to be stable N≥M**.

2.6Filter specification

There is several parameters and definitions of filters must be illustrated before talking about filters types or filters design, so here the main parameters and specification will be illustrated and explained.

2.6.1 Filter'sorder

The order of a filter is important for severalreasons. It is directly related to the number of componentsin the filter, and therefore to its cost, its physical size, andthe complexity of the design task.

Therefore, higher-order filters are more expensive, take up more space, and aremore difficult to design. The primary advantage of a higherorderfilter is that it will have a steeper rolloff slope than asimilar lower-order filter.

We will show in the next chapter that, the degree of the filter is related with the number of the reactive elements in the circuit. Figure 2.5 shows that; by increasing the degree of the filter the response of the filter will be sharper, or in other word the selectivity of the filter will be increased.

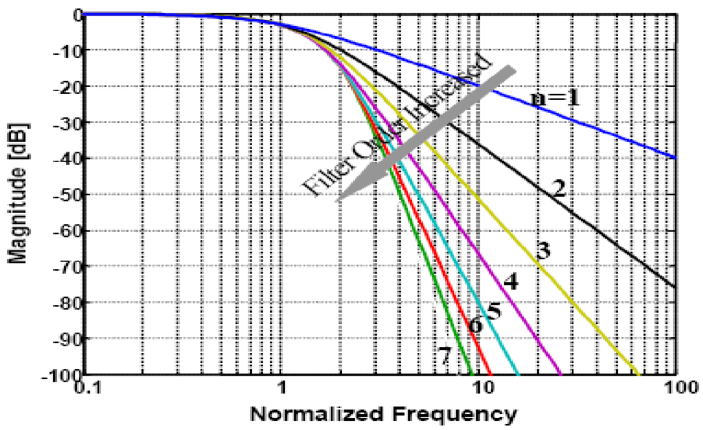

Fig 2.5 : ***LPF response for several degrees.***

2.6.2 ideal and practical filter

An ideal filter is a network that allows signals of only certain frequencies to pass while blocking all others. Depending on the regime of frequencies that are allowed through or notthey are characterized as low-pass, high-pass, band-pass, band-reject and all-pass.. The frequency response is divided into magnitude(amplitude) and phase parts. The amplitude curve of a filter will indicate how closely thepractical circuit imitates the ideal filter characteristics that are as follows:

These ideal characteristics will at best be approximated by real circuits. How closely this will be achieved will depend on the frequency response of the particular circuit.

A low pass filter is a circuit whose amplitude (magnitude) function decreases asωincreases, that is, the circuit passes low frequencies (relatively large amplitudes at theoutput) and rejects high frequencies (relatively small amplitudes at the output) asshown in fig 2.6

In fig 2.6, ω_c is defined as the (3 dB) frequency, that is the frequency at which theamplitude is (1/2)1/2 = 0.707 times the maximum amplitude. It is traditional to consider

the 3 dB frequency as the cutoff frequency. That is, a low pass filter is said to passfrequencies lower than !c and reject those that are higher than !c. In other words, thepass(ing) band is $\omega<\omega_c$.

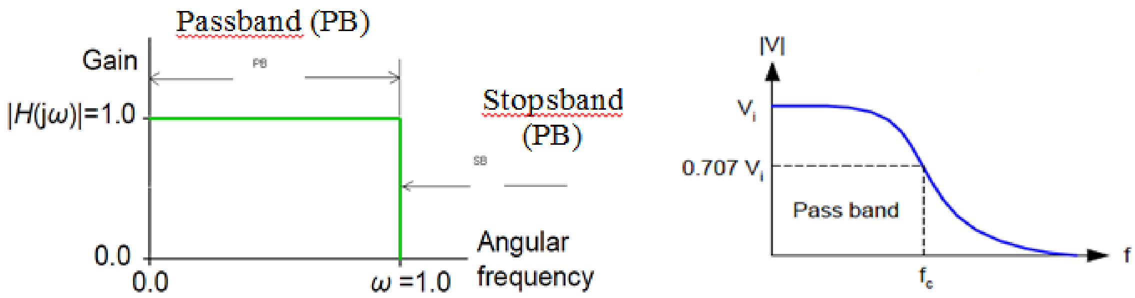

Fig 2.6 : response of ideal and practical filter.

2.6.3 Passive and active

Passive, which indicates that there are no active elements in the filter implementation, but usually also is further restricted to an implementation that is made up of R's, L's and C's. Filters realized with RLC circuits have limited performance. Filters incorporating suchpassive elements in addition to active elements such as op amps exhibit bettercharacteristics and are called active filters.

Active, which indicates that theimplementation includes active elements, such as operational amplifiers (op amps), orpossibly other active elements such as transistors. The main advantage of op ampactive filters, due to the very low output impedance characteristic of op amps, and alsovery high open loop gain, and high input impedance, is that op amp stages haveinherent buffering, which means that the overall transfer function of several op ampstages is the product of the individual stage transfer functions, ignoring loading effectsof subsequent stages.

2.7 Filter parameters

2.7.1 $F_o$and Q

The groundwork has been set for two concepts that will be infinitelymore useful in practice: F_o and Q.

F_o is the cutoff frequency of the filter. This is defined, in general, as the frequency wherethe response is down 3 dB from the pass band. It can sometimes be defined as thefrequency at which it will fall out of the pass band. For example, a 0.1 dB Chebyshevfilter can have its F_o at the frequency at which the response is down > 0.1 dB.

The shape of the attenuation curve (as well as the phase and delay curves, which definethe time domain response of the filter) will be the same if the ratio of the actual frequencyto the cutoff frequency is examined, rather than just the actual frequency itself.

Normalizing the filter to 1 rad/s, a simple system for designing and comparing filters canbe developed. The filter is then scaled by the cutoff frequency to determine thecomponent values for the actual filter.

Q is the “quality factor” of the filter. It is also sometimes given as α where:

$$\propto = \frac{1}{Q} \qquad [11]$$

This is commonly known as the damping ratio. ξ is sometimes used where:

$$\xi = 2 \propto \qquad [12]$$

If Q is > 0.707, there will be some peaking in the filter response. If the Q is < 0.707,rolloff at F_0 will be greater; it will have a more gentle slope and will begin sooner. Theamount of peaking for a 2 pole low-pass filter vs. Q is shown in Figure 2.7.

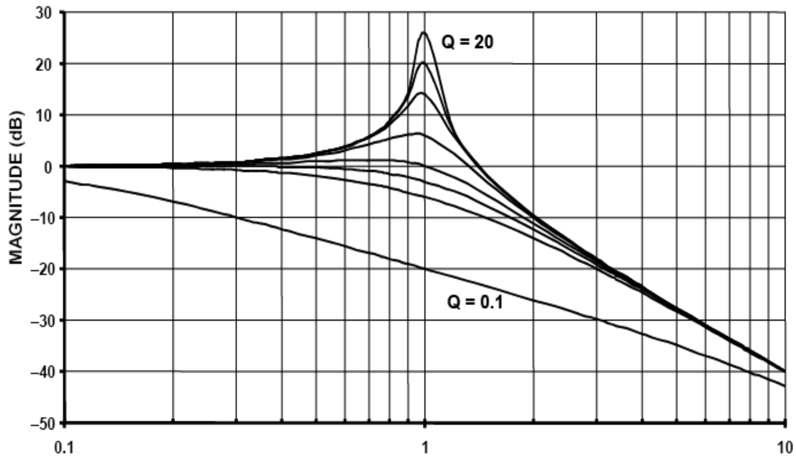

Fig 2.7 : Theamount of peaking for a 2 pole low-pass filter vs. Q.

2.7.2 Response parameters

Refer to Figure 2.8 as an aid in defining these bands for lowpass filter. Figure shows the vertical axis is in dB, and the passband edge ω_c is and the stopband edge is ω_s.

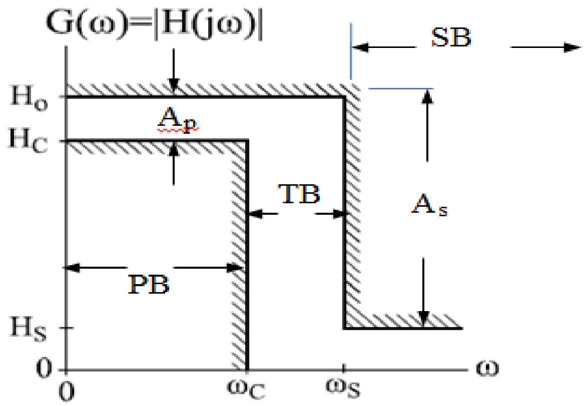

Fig 2.8 : Filter response parameters

As shown in the fig 1.9. The maximum passband gain has been normalized to 0 dB, and the **passband gain** is acceptable as long as it is within the range of 0 dB and $-A_p\, dB$.

The value of $-\boldsymbol{A_p}$ is called **passband attenuation**, it is a small value, e.g. 1 dB.

The value of**Stopband rejection $\boldsymbol{A_s}$** is the specified attenuation in the stopband, e.g., 60 dB.

Definitions of passband, stopband, and transition band are as follows:

Passband (PB)

The passband is the range of frequencies over which the magnitude response does not fall below $-A_p$ .For a lowpass filter, the passband is from $\omega = 0$ to$\omega = \omega_c$. For a highpass filter, the passband is from$\omega = \omega_c$to $\omega = \infty$. For a bandpass filter, the passband is from$\omega = \omega_{c1}$to $\omega = \omega_{c2}$. For a bandstop filter there are two passbands, from $\omega = 0$ to $\omega = \omega_{c1}$ and from $\omega = \omega_{c2}$ to$\omega = \infty$.

Stopband (SB)

The stopband is the range of frequencies over which the magnitude response does not rise above A_s. For a lowpass filter, the stopband is from $\omega = \omega_c$ to $\omega = \infty$. For a highpass filter, the stopband is $\omega = 0$ from to $\omega = \omega_c$. The stopband is defined in similar ways for bandpass and bandstop filters.

Transition Band (TB)

The transition band is the range of frequencies between the passband and the stopband. For a lowpass filter, the transition band is from $\omega = \omega_s\ to\ \omega = \omega_c$.

In order to predict the behavior of an operational amplifier when circuit elements areexternally connected to its terminals, one must understand the constraints imposed on the

terminal voltages and currents by the amplifier itself. Those imposed on the terminalvoltages are as follows :

$$V_0 = A_V (V_p - V_n) \quad [13]$$

$$V^- = -V_{cc} \leq V_0 \leq +V_{cc} = V^+ \quad [14]$$

Eq. 16 states that the output voltage is proportional to the difference between V_p and V_n. If the output voltage V_o is to be finite, it follows from the definition of voltage gain, that $Vi = V_o / A_v$ will go to zero when Av is infinite. This, however, assumes that there is someway for the input to be affected by the output. Indeed this will only happen if there isnegative feedback in the form of a connection between the output and the invertingterminal (closed loop operation).

For closed loop operation, it is said that a virtual shortexists between the inverting and noninverting input terminals. This means that if an OpAmp is operating in its linear region (if it is unsaturated) then Vi = 0, or equivalently $V_n = V_p$.

Eq. 2 states that the output voltage is bounded. In particular, V0 must lie between ±VCC, thepower supply voltages. Else V_0 will be at either limiting value, and the Op-Amp is thensaturated. The amplifier is operating in its linear range so long as |V0| < |VCC|.

Chapter 3

FILTERS TYPES

3.1 introduction

There are many ways in which to classify filters, it cans classes from several points of view, one of them their behavior at frequency domain, the other is their response shape, some others to the structure of the filter.

3.2 Main types of filters

When structure of the system is taken in account, filters can be classified into digital and analog filters.

3.2.1 Digital filters

Often, digital filters are used to process analog signals byfirst going through an analog-to-digital converter. After processing, the output of thedigital filter may well then be converted back to an analog signal. In such a real-timefiltering situation, usually accomplished with a microprocessor, and commonly witha microprocessor designed especially for signal processing applications, the filteringapplication is analog-in and analog-out. However, digital filtering is also oftenaccomplished off-line in personal or mainframe computers.

3.2.2 Analog filters

Analog filters may be referred to as **passive**, which indicates that there are no active elements in the filter implementation, but usually also is further restricted to animplementation that is made up of R's, L's and C's. Analog filters that are alsopassive, but more specialized, would include mechanical resonators and quartz crystal filters.

Analog filters may also be referred to as **active**, which indicates that theimplementation includes active elements, such as operational amplifiers (op amps), orpossibly other active elements such as transistors. The main difference between passive filters and active filters (apart from the active filter's ability to amplify signals) is that active filters can produce much steeper cut off slopes. However, passive filters do not require any external power supply and are adequate for a great many uses.

The main advantage of op ampactive filters, due to the very low output impedance characteristic of op amps, and alsovery high open loop gain, and high input impedance, is that op amp stages haveinherent buffering, which means that the overall transfer function of several op-ampstages is the product of the individual stage transfer functions, ignoring loading effectsof subsequent stages. This greatly simplifies the theoretical implementation.

That is,for example, a sixth-order op-amp filter can be implemented by cascading threesecond-order op-amp stages, where each second-order stage is implementedindependently of the other two stages. Passive analog filters do not enjoy thissimplification, and the entire transfer function must be implemented as one nonseparablewhole. In this thesis, we will study and design only passive filters.

3.3 Basic types of frequency-selective Filters

There are five basic filter types (bandpass, notch, low-pass,high-pass, and all-pass).

Low-pass filters allow any input at a frequency below acharacteristic frequency to pass to its output unattenuated oreven amplified.

High-pass filters allow signals above a characteristic frequency topass unattenuated or even amplified.

Band-pass filters allow frequencies in a particular range to passunattenuated or even amplified.

Notch, low-passprevent(attenuate) frequencies in a particular range to pass.Fig 3.1 shows the frequency response for the last types

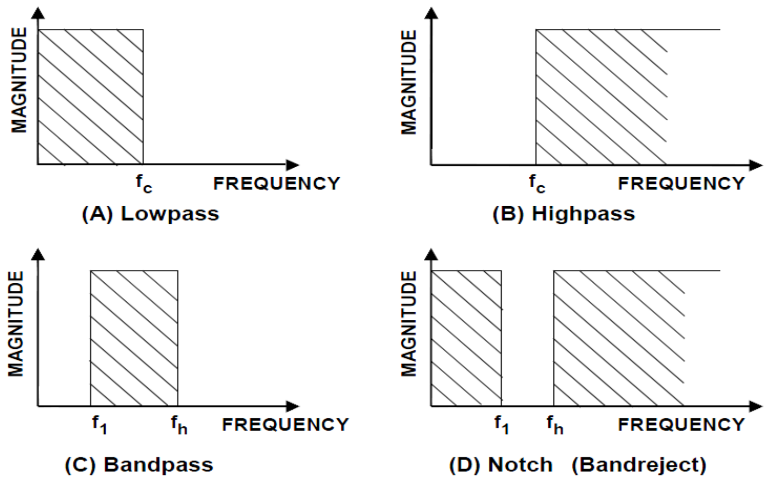

Fig 3.1 : frequency response for basic filters types.

3.3.1 Low-Pass (LPF)

A low-pass filter passeslow frequency signals, and rejects signals at frequenciesabove the filter's cutoff frequency.

Low-pass filters are used whenever high frequency componentsmust be removed from a signal. An example might bein a light-sensing instrument using a photodiode. If light levelsare low, the output of the photodiode could be verysmall, allowing it to be partially obscured by the noise of thesensor and its amplifier, whose spectrum can extend to veryhigh frequencies. If a low-pass filter is placed at the outputof the amplifier, and if its cutoff frequency is high enough toallow the desired signal frequencies to pass, the overallnoise level can be reduced.

Various approximations tothe unrealizable ideal low-pass amplitude characteristicstake different forms as will show next, some being monotonic (always havinga negative slope), and others having ripple in the passbandand/or stopband. Figure 3.2 shows butterworth response of practical LPF.

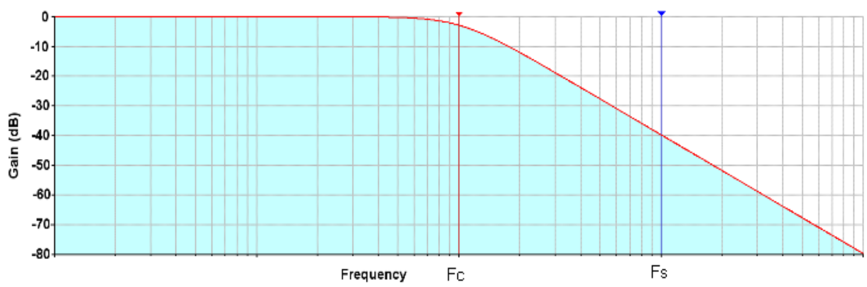

Fig 3.2 : The frequency response of low-pass filter.

All low-pass filters are rated at a certain cutoff frequency F_c. That is, the frequency above which the output voltage falls below 70.7% of the input voltage. This cutoff percentage of 70.7 is not really arbitrary, all though it may seem so at first glance.

3.3.2 High Pass Filter (HPF)

High-pass filters are used in applications requiring the rejectionof low-frequency signals. One such application is inhigh-fidelity loudspeaker systems. Music contains significantenergy in the frequency range from around 100 Hz to 2 kHz,but high-frequency drivers (tweeters) can be damaged iflow-frequency audio signals of sufficient energy appear attheir input terminals. A high-pass filter between the broadbandaudio signal and the tweeter input terminals will preventlow-frequency program material from reaching thetweeter.

.A high-pass filter's task is just the opposite of a low-pass filter as shown in figure (3.3): to offer easy passage of a high-frequency signal and difficult passage to a low-frequency signal.

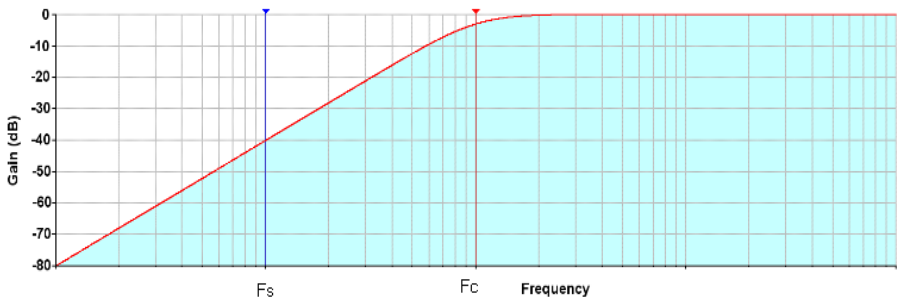

Fig 3.3 : The frequency response of high-pass filter.

As with low-pass filters, high-pass filters have a rated *cutoff frequency*, above which the output voltage increases above 70.7% of the input voltage.

3.3.3 Bandpass pass filters (BPF)

Bandpass filters are used in electronic systems to separatea signal at one frequency or within a band of frequenciesfrom signals at other frequencies. In figure 2.4 an example wasgiven of a filter whose purpose was to pass a desired signalat frequency f_1, while attenuating as much as possible anunwanted signal at frequency f_2. This function could be performedby an appropriate bandpass filter with center frequencyf1.

Such a filter could also reject unwanted signals atother frequencies outside of the passband, so it could beuseful in situations where the signal of interest has beencontaminated by signals at a number of different frequencies.

The number of possiblebandpass response characteristics is infinite, but they allshare the same basic form. an examples of bandpassamplitude response curves are shown in Figure 3.4. is examples of bandpass amplitude responsecurves that approximate the ideal curves with varyingdegrees of accuracy.

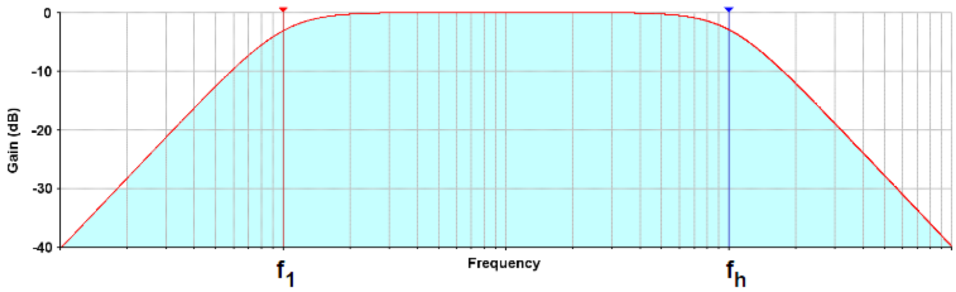

Fig 3.4 : ***Bandpass amplitude response curve.***

Bandpass filtershave two stopbands, one above and one below the passband.Just as it is difficult to determine by observation exactlywhere the **passband ends**, the boundary of the stopband isalso seldom obvious. Consequently, **the frequency at which a stopband begins is usually defined by the requirements of a given system-** for example, a system specification mightrequire that the signal must be attenuated at least 35 dB at1.5 kHz. This would define the beginning of a stopband at1.5 kHz.

The rate of change of attenuation between the passbandand the stopband also differs from one filter to the next.

The slope of the curve in this region depends strongly on the order of the filter, with higher-order filters having steepercutoff slopes. The attenuation slope is usually expressed indB/octave (an octave is a factor of 2 in frequency) or dB/decade (a decade is a factor of 10 in frequency).

3.4 Filter Specifications and Approximations

The behavior of analogue filters can be described in the time or in the frequency domain and filters canbe designed from time or frequency domain specifications.

However, it is more often the case that filtersare designed from frequency domain specifications from either amplitude or phase requirements.

Filter specifications are usually given in terms of magnitude characteristics and describe the desirablegain or attenuation in the passband and stopband within specified tolerances. Thecomplexity of the transfer function of the filter and of the filter itself depends heavily onthese tolerances and increases as these become stricter.

All filter approximations, tables and nomographs refer to normalized lowpass specifications since they are not only easily denormalized but can be transformed to any other filter type,such as highpass, band-reject and bandpass filters as we will explain next.

Approximations are mathematical procedures used to translate given magnitude pecifications intorealizable transfer functions. Employment of well known approximations like Butterworth, Chebyshev,elliptic etc. to determine the transfer function of a filter ensures that the magnitude response willsatisfy the specifications and will be realizable with passive or active circuits.

The synthesis of a normalized lowpass filter with given magnitude specifications starts with findinga gain function) that satisfies the specifications with $G(\omega)^2$ being a rational even functionof ω. This procedure, the approximation, yields theoretically infinite solutions and some of these arenot suitable since they do not satisfy certain realizability conditions. This means that simply findinga mathematical function, the plot of which does not violate the specifications, does not necessarilyassure its realizability as a gain function of a realizable circuit.

The filter designer can choose from some establishedapproximations that lead to realizable transfer functions. Among these, the most popular are theButterworth, Chebyshev and Cauer (or elliptic) approximations.

The manner in which each of themapproximates the ideal lowpass gain specifications is shown in Fig. 3.5. From these approximations, Butterworth, Chebyshev and Pascal, (a), (b) and (d) in Fig. 3.1 are monotonic in the stopband. Inverse Chebyshev (Fig. 3.5c), inverse Pascal (Fig. 3.5e) and Cauer (Fig. 3.5f) have transmission zeros, i.e. frequencies at which the plain gain becomes zero.

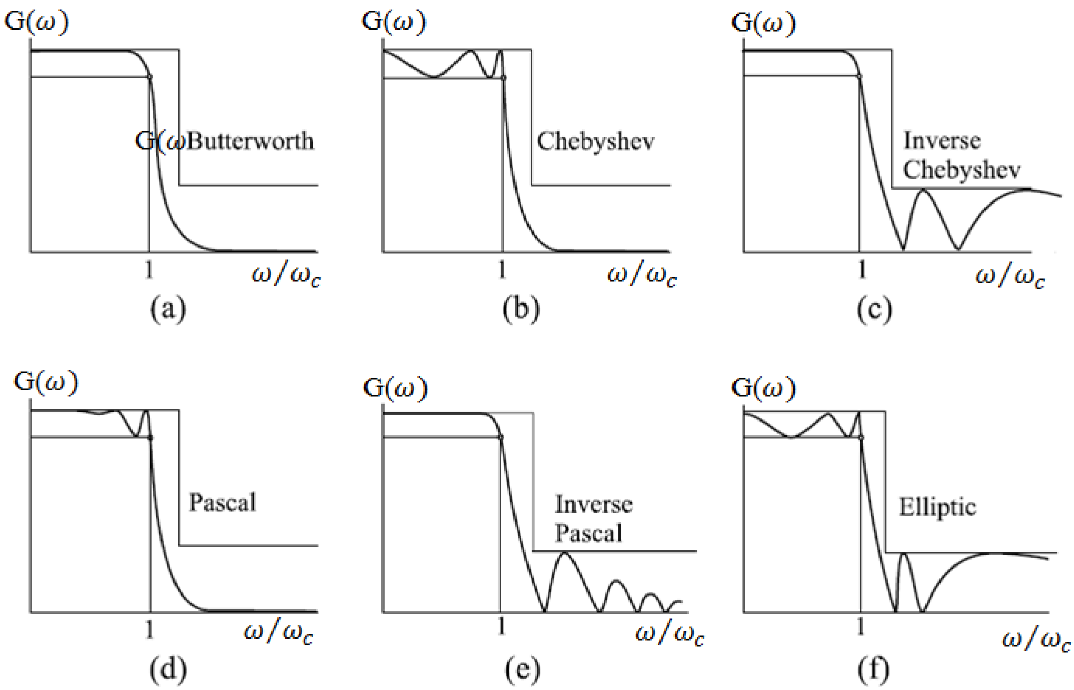

Fig 3.5 : Butterworth, Chebyshev, Pascal and elliptic approximations.

In order to synthesize and finally implement a passive or active filter from magnitude specifications,it is necessary to calculate its transfer function

$$H(s) = \frac{X_{out}(s)}{X_{in}(s)} \quad [15]$$

The approximation gives the gainfunction G(ω) = $|H(s)|_{s=j\omega}$. The poles and zeros of the required and stable transfer function areobtained by a factorization process, the basis of which is as indicated the equation.

$$H(s)H(-s) = |H(j\omega)^2| \quad [16]$$

The approximation procedure is completed when we have constructed the transfer function H(s), themagnitude of which $|H(s)|_{s=j\omega}$ = G(ω) satisfies the filter specifications.

The specifications for a low-pass filter are often stated to require the magnitude of the filter frequency response to lie in the nonshaded area indicated in Figure 3.6.

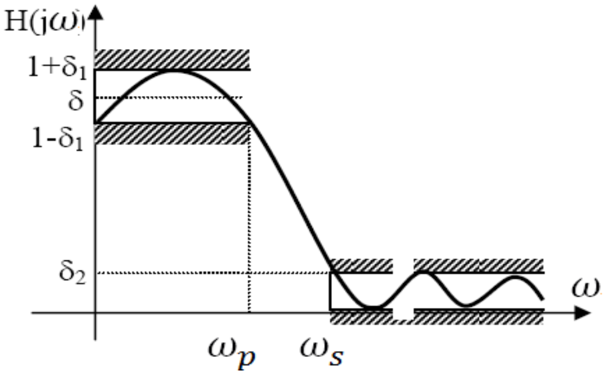

Fig 3.6 : General frequency response of LPF

In theabove figure a deviation from unity of $\pm\delta_1$ is allowed in the pass-band and a deviation of δ_2 zero is allowed in the stop-band. The amount by which the frequency response differs from unity in the pass-band is referred to as the *pass-band ripple* and the amount by which it deviates from zero in the stop-band is referred to as the *stop-band ripple*. The frequency ω_p, is referred to as the *pass-band edge* and ω_s, as the *stop-band edge*. The *transition band* $\Delta\,\omega = \omega_s - \omega_p$provides the transition from pass-band to stop-band.

It is evident that the ideal low-pass filter is noncausal and consequently must be approximated for real-time filtering by a causal system. When filtering is to be carried out in real time, causality is a necessary constraint, and thus a causal approximation to the ideal characteristics would be required. A further consideration that motivates providing some flexibility in the filter characteristics is ease of implementation.

Analog filter design is often based on the use well-known models called Butterworth,Chebyshev and Bessel filters.

3.4.1 Butterworth Approximation

The magnitude-squared response of an analog lowpass Butterworth filter aH (s) of Nth orderis given by,

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\frac{\omega}{\omega_c})^{2N}}}2 \qquad [17]$$

It can be easily shown that the first 2N-1 derivatives of $|H(j\omega)|$at ω= 0 is equal to zero, and as a result, the Butterworth filter is said to have a maximally-flat magnitude at ω= 0. The gain of the Butterworth filter in dB is given by,

$$H(\omega) = 10\, log \log_{10}(|H(j\omega)|)^2\ dB. \quad [18]$$

At ω= 0, the gain in dB is equal to zero, and at ω= ωc, the gain is,

$$H(\omega_c) = 10 \log_{10}\left(\frac{1}{2}\right) = -3.0103 \cong -3\, dB$$

and therefore, It is often called the 3-dB cutoff frequency. Since the derivative of the squaredmagnitude response, or equivalently, of the magnitude response is always negative for positive values of ω, the magnitude response, is monotonically decreasing with increasing ω.

For $\omega >> \omega c$, the squared-magnitude function can be approximated by,

$H(\omega) = 10\, log \log_{10}(|H(j\omega)|)^2$The gain H($\omega$2) in dB at $\omega_2 = 2\omega_1$with $\omega_1 >> \omega_c$

The two parameters completely characterizing a Butterworth filter are therefore the 3-dBcutoff frequency $\omega_c$and the order N. These are determined from the specifiedpassband edge ω_p, the minimum passband magnitude$1/\sqrt{1+\varepsilon^2}$, the stopband edgeω_S, andthe maximum stopband ripple 1/A. From Eq. (17) we get,

$$|H(j\omega)|^2 = \frac{1}{1+(\omega/\omega_c)^{2N}} = \frac{1}{1+\varepsilon^2} \quad [19]$$

Since the order N of the filter must be an integer, the value of N computed using the above expression is rounded up to the next higher integer. This value of N can be used next in either use to solve for the 3-dB cutoff frequencyω_C.Fig 3.7 shows the relation between frequency response and the filter degree N.

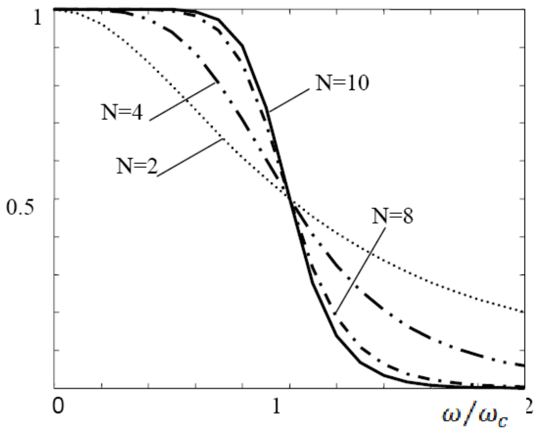

Fig 3.7 : Buttrworth response as a function to N.

3.4.2Chebyshev Approximation

In some applications, the sharpness of the cutoff response is more important than the passband flatness. By adding higher resonant peaks, it is possible to obtain sharper cutoff at the expense of peaks in the passband. In this case, the approximation error, defined as the difference between the ideal brickwall characteristic and the actual response, is minimized over a prescribed band of frequencies. In fact, the magnitude error is equiripple in the band. There are two types of Chebyshev transfer functions. In the Type 1 approximation, the magnitude characteristic is equiripple in the passband and monotonic in the stopband, whereas in the Type 2 approximation, the magnitude response is monotonic in the passband and equiripple in the stopband. In this study only Type 1 approximation will be illustrated.

Whereas for the Butterworth filter, we only specify the number of poles or zeroes of the filter, for a Chebyshev filter, we specify the number of poles (zeroes) and passband flatness (i.e., a 0.5dB Chebyshev filter has a minimum peak 0.5dB above the minimum valley in the passband (equal–ripple filter). So the amount of passband ripple is one of the parametersused in specifying a Chebyshev filter.

The Chebyschevcharacteristic has a steeper roll-off near the cutoff frequencywhen compared to the Butterworth, but at the expenseof monotonicity in the passband and poorer transientresponse. A Chebyshev filter responses areshown in Figure 28. The filter responses in the figure have a ripple in the passband equal to A_pdB.

A Chebyshev filter of order n has n-1 peaksor dips in its passband response. The nominalgain of the filter (unity in the case of the responses inFigure2.8) is equal to the filter's maximum passband gain. Also figure 3.8 shows the representative transmission functions for Chebyshev filters of even and odd orders.

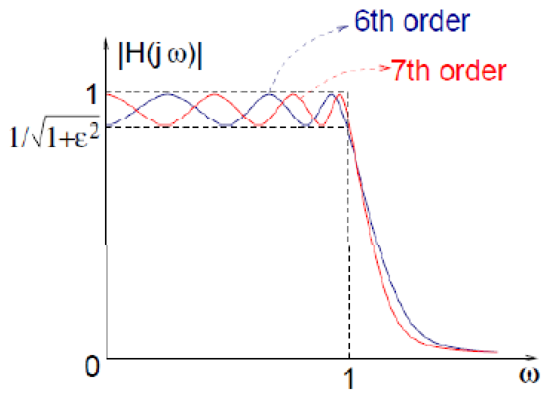

Fig 3.8 : transmission functions for Chebyshev filters of even and odd orders.

An odd-orderChebyshev will have a dc gain (in the low-pass case)equal to the nominal gain, with "dips" in the amplitude responsecurve equal to the ripple value. An even-orderChebyshev low-pass will have its dc gain equal to thenominalfilter gain minus the ripple value; the nominal gain for aneven-order Chebyshev occurs at the peaks of the passbandripple. Therefore, if we're designing a fourth-order Chebyshevlow-pass filter with 0.5 dB ripple and we want itto have unity gain at Dc,we'll have to design fora nominalgain of 0.5 dB.

The cutoff frequency of a Chebyshev filter is not assumed tobe the -3 dB frequency as in the case of a Butterworthfilter. Instead, the Chebyshev's cutoff frequency is normallythe frequency at which the ripple specification isexceeded.

The addition of passband ripple as a parameter makes thespecification process for a Chebyshev filter a bit more complicatedthan for a Butterworth filter, but also increases flexibility. All the transmission zeroes of the Chebyshev filter are at$\omega=\infty$ making it an all pole filter.So the chebychev approximation is an all-pole response as the Butterworth approximation.

At the passband edge $\omega = \omega_p \quad |H(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2}}$ [20]

The parameter ε determines the passband ripple according to:

$$A_{p\,max} = 20\,log\sqrt{1+\varepsilon^2} \qquad and \qquad \varepsilon = \sqrt{10^{A_{max}/10}-1} \qquad [20]$$

Chapter 4
Passive filters design

4.1 Introduction

In principle, filters can be made from passive components, that is resistors,capacitors and inductors. However, at low frequencies, typically below100MHz, the inductors required to generate a reasonable impedanceare bulky. Furthermore, they will include significant resistance that willlimit the performance of any filter. Most filters therefore containresistors and capacitors.

The simplest passive filter can be created using a resistor andcapacitor. An example of this type of circuit designed to be a low-passfilter is shown in Figure (4.1 (a)).

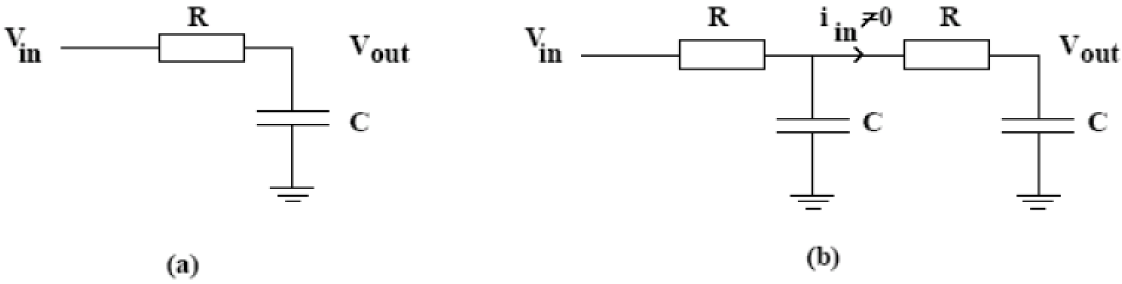

Fig 4.1 : Passive filter created using a resistor andcapacitor

The response of this type of filter, is

$$G(\omega) = \frac{V_{out}}{V_{in}} = \frac{1/j\omega c}{R + 1/j\omega c} \qquad [21]$$

Which can be re-written as

$$G(\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega Rc} \qquad [22]$$

to show that the circuit attenuates high frequency signals. Unfortunately, thefrequency dependence of these simple filters is relatively weak. Such filtersmay be adequate when an unwanted interfering signal is far removed infrequency from the desired signal.

If a sharper response is required, the simple solution would appear tobe to connect several simple RC filters in series, see for example the twocircuits in Figure (4.1 (b)). In this situation the input signal to each stageis the output signal from the previous stage and it might be expectedthat the response of n identical RC circuits connected in series wouldbe

$$G_n(\omega) = G(\omega)^n \qquad [23]$$

However, this simple approach to calculating the response of thissystem will give an incorrect answer. This failure arises because thissimple calculation ignores the current that each circuit draws from theprevious filter.

This problem can be solved by inserting an op-amp voltage followerbuffer between the various filters as shown in Figure (4.2).

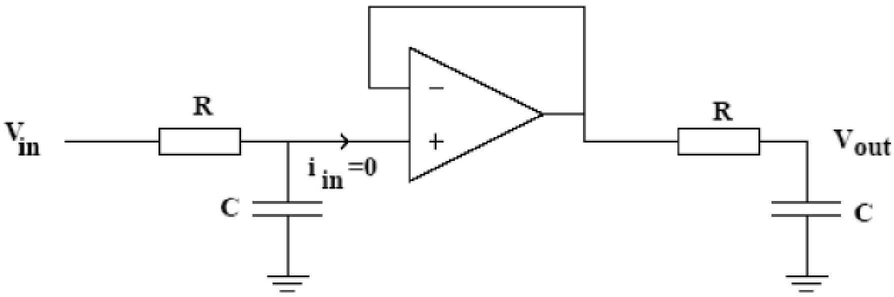

Fig 4.2 : Inserting an op-amp buffer between two filters.

Now theinfinite input impedance of the ideal op-amp means that the currentdrawn from each RC circuit is zero, as assumed in the analysis of thesingle RC circuit, andcan be used to design filters with sharper roll-offs. The sharper roll-offof the high frequency response resulting from connecting two and threefilters in series is clearly shown in Figure (4.3).

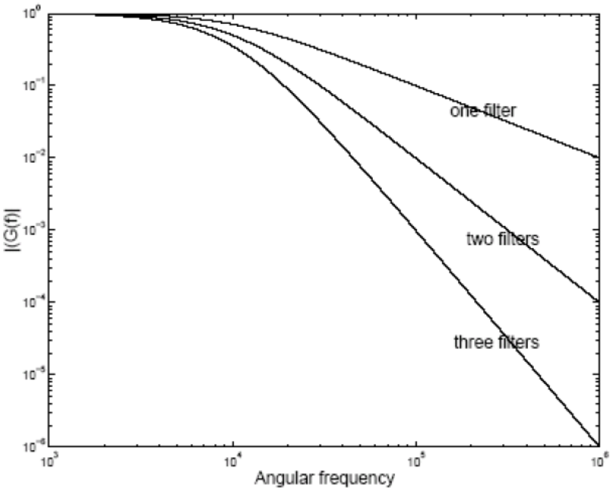

Fig 4.3 : Increasing selectivity by increasing filters number.

The problem with these passive filters is that frequency selectivity isachieved by dissipated unwanted frequencies and it is impossible forany frequency components in the input signal to be amplified by thistype of filter.

4.2 Passive filters types:

There are three basic kinds of circuits capable of accomplishing this objective, and many variations of each one: **The inductive low-pass filter** in Figure 4.4-A and the **capacitivelow-pass filter** in Figure 4.4-B, and **resonant low-Pass filter**in Figure 4.4-C

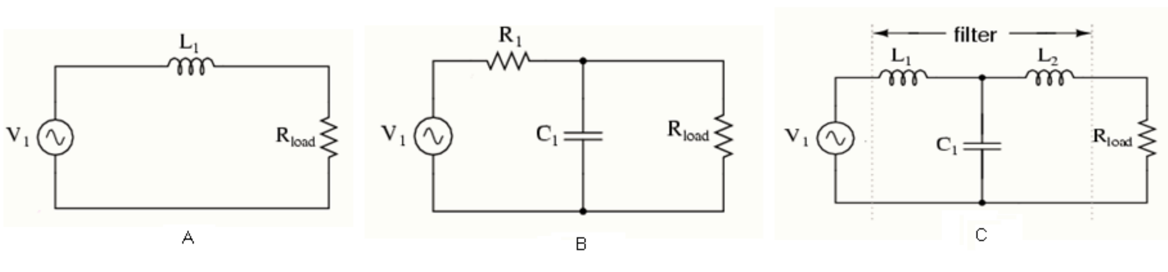

Fig 4.4 : LPF types

To indicate the effect a filter has on wave amplitude atdifferent frequencies, a frequency response graph is used.

Passive filters only contain components such as resistors, capacitors, and inductors. This means that, the signal amplitude at a filter output cannot be larger than the input. The maximum gain on any of the frequency response graphs is therefore slightly less than 1.

All low-pass filters are rated at a certain cutoff frequencyf_c. That is, the frequency above which the output voltage falls below 70.7% of the input voltage. This cutoff percentage of 70.7 is not really arbitrary, all though it may seem so at first glance. In a simple capacitive/resistive low-pass filter, it is the frequency at which capacitive reactance in ohms equals resistance in ohms. In a simple resonance low-pass filter; the cutoff frequency is given as:

$$\omega_c = \frac{1}{\sqrt{LC}} \quad [24]$$

As one might expect, the inductive and capacitive versions(as shown in figure 4.5) of the high-pass filter are just the opposite of their respective low-pass filter designs.

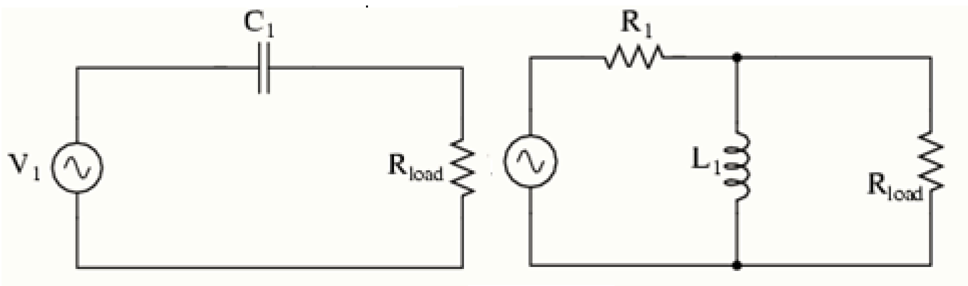

Fig 4.5 : Capacitive andinductivehigh-pass filter.

The inductor's impedance decreases with decreasing frequency. This low impedance in parallel tends to short out low-frequency signals from getting to the load resistor. As a consequence, most of the voltage gets dropped across series resistor R1.

As with low-pass filters, high-pass filters have a rated *cutoff frequency,* above which the output voltage increases above 70.7% of the input voltage. Just as in the case of the capacitive low-pass filter circuit, the capacitive high-pass filter's cutoff frequency can be found with the same formula:

$$f_{cutoff} = \frac{1}{2\pi RC} \quad [25]$$

While in resonant circuit cutoff frequency will be equal to

$$f_{cutoff} = \frac{1}{2\pi\sqrt{LC}}[26]$$

4.3 Design procedure for analog filter

The design procedure for analog filter has several distinct stages, in the first stage a frequency response function $|H(j\omega)|$is derived which meet the specification, which based upon the application of the filter. The specifications generally include A_p, A_s, $\omega_s$andω_c, if it is a lowpass or highpass filter, or similar specifications if it is a bandpass, or bandstop filter.

The specifications will likely also include the filter type, such as Butterworth,Chebyshev... etc, also the degree of the filter must be concentrated.

4.3.1 Design lowpass prototype filter

In the second stage an electronic circuit is designed to generate the required frequency function. Lowpass filter prototype can be obtained using a table procedure as one of main procedures. Tables (1) can be used to find the values of resistors, capacitors and inductors for various Chebyshev and Butterworth filters. Usually, element values in tables are normalized for a critical frequency of unity, and for either a source resistor or load resistor of unity.

Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

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Table (1) normalized values of circuit component for Butterworth filters

Element Values for Equal-Ripple Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10, 0.5 dB and 3.0 dB ripple)

0.5 dB Ripple											
N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

3.0 dB Ripple											
N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

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Tables (2) normalized values of circuit component for various Chebyshev and filters

The filters are design to have an impedance matching between the source and the load to obtain maximum power transfer. The given values are normalized (i.e., resistance 1Ω and frequency 1rad/sec), the passive circuit can be designed in ladder form as in figure 4.6, designer can choose one of two shapes, π shape as in figure 4.6-a or T shape as in figure 4.6-b.

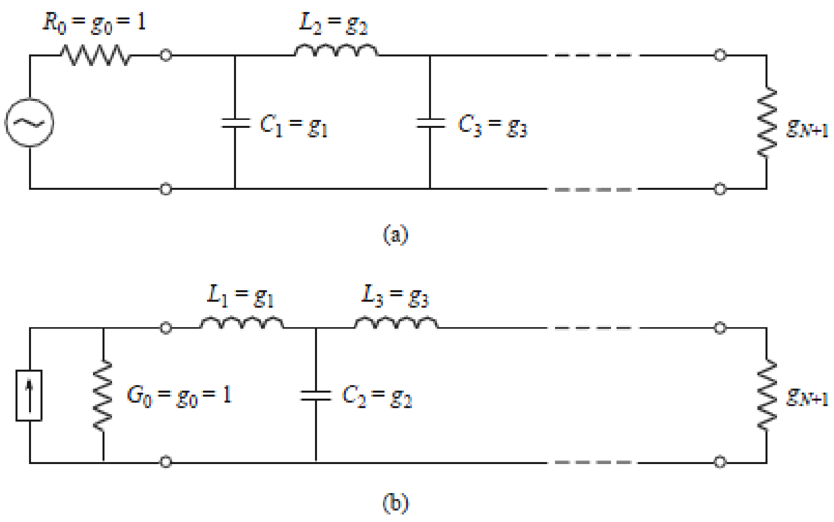


Fig 4.6 : Ladder circuit for low-pass filter and their element definitions

4.3.2 FILTER TRANSFORMATIONS

The low-pass filter prototypes of the previous section were normalized designs having asource impedance of R_s = 1 Ω and a cut-off frequency of ω_c = 1 rad/sec. Here we will showhow these designs can be scaled in terms of impedance and frequency, and converted togive high-pass, bandpass, or bandstop characteristics.

a- Impedance and Frequency Scaling

Impedance scaling:

In the prototype design, the source and load resistances are unity (exceptfor equal-ripple filters with even N, which have nonunity load resistance). A sourceresistance of R_0 can be obtained by multiplying all the impedances of the prototype designby R_0. Thus, if we let primes denote impedance scaled quantities, the new filter componentvalues are given by designers have to scale the components to fit the requirements of the system according to the following guide lines:

Let

R_o = system impedance (e.g 50 Ω or 75Ω).

$\omega_c = 2\pi f_c$ (Where fc is the cut off frequency).

$R', \; L', C'$the normalized values from the table.

Then we can calculate the real values of passive electronic components from relations:

$$\left.\begin{aligned} R_{s} &= R_o \\ R_L{}' &= R_o R_L \\ C' &= \frac{C}{R_o}, \\ L' &= R_o L, \end{aligned}\right\} \qquad [27]$$

Frequency scaling for low-pass filters:

To change the cutoff frequency of a low-pass prototypefrom unity to ω_c requires that we scale the frequency dependence of the filter by thefactor $1/\omega_c$, which is accomplished by replacing ω by ω/ω_c.

$$\omega \leftarrow \frac{\omega}{\omega_c}.$$

So denormalized values of filter can be calculated from the following relations

$$\left.\begin{aligned} R_{d} &= R_o R_{N}, \\ L_{d} &= \frac{R_o}{\omega_c} L_N \\ C_{d} &= \frac{1}{R_o \omega_c} C_{N}, \end{aligned}\right\} \qquad [28]$$

The design may be obtained by use of MATLAB, using functions that come with MATLAB. This design procedure would likely also include detailed analysis of the proposed design, to reveal as much detail as possible about the proposed design's performance prior to implementation.

b- Bandpass and Bandstop Transformations

We declared in last sections how we can design Lowpass filter prototype, now if we want to design HPF or BPF, frequency transformations must be applied directly to the element values of the lowpass prototypeimplementation. In addition, impedance scaling (reviewed in this chapter) may beapplied for desirable element values. Figure 4.7 Declare the mean relation which usually used to apply filter transformations:

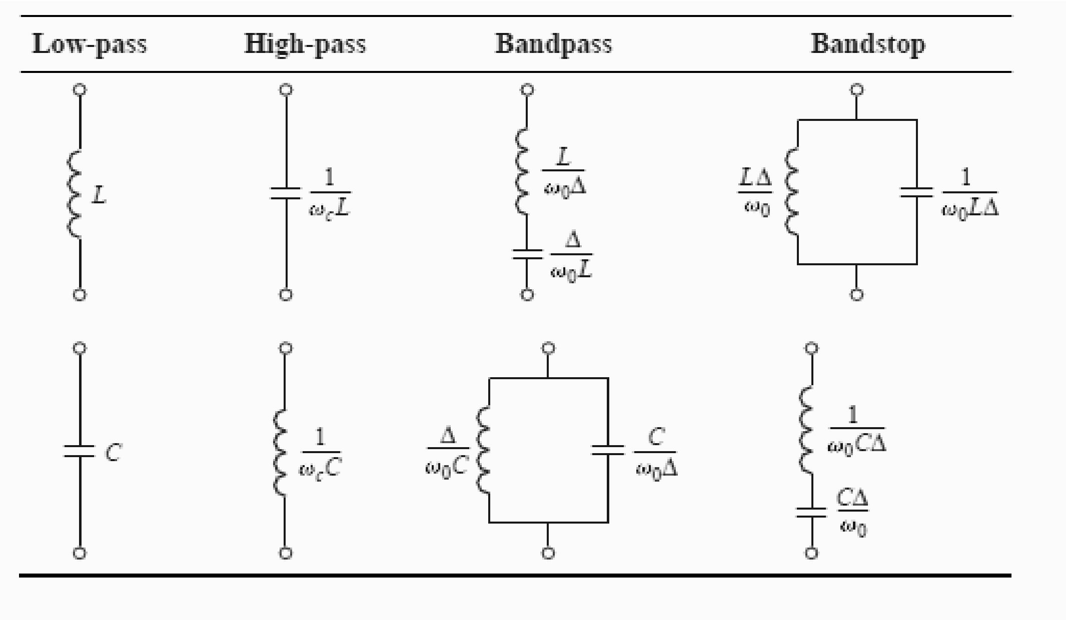


Fig 4.7: : summary of prototype filter transformations $\left(\Delta = \frac{\omega_2 - \omega_1}{\omega_o}\right)$

$$\omega \leftarrow \frac{\omega_0}{\omega_2 - \omega_1}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

$$\Delta = \frac{\omega_2 - \omega_1}{\omega_0}$$

Therefore, be using tables method, designer can design LPF prototype, after he indicating the filter speciffications, then he can transfer his design to the wanted type (HP or BP … filters….etc).

Chapter 5
Active filters design

5.1Introduction

This studywill illustrate active filters, which use op-amp as an active component, here main characteristics of op-amps will be reviewed. Fig. 5.1 shows main parameters of an operational Amplifier.

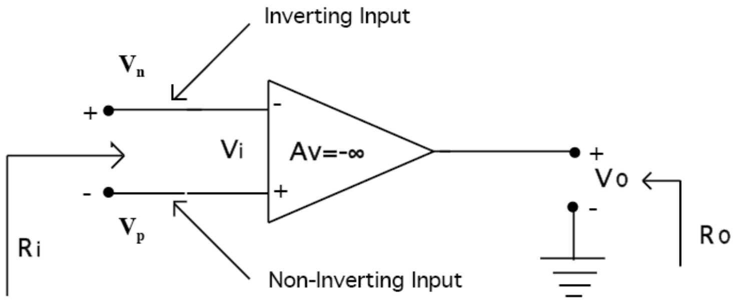


Fig 5.1 : main parameters of an operational Amplifier.

Characteristics of an Ideal Op Amp

1. Input Resistance Ri = ∞: An infinite inputresistance means that no current flows intoor out of either of the input terminals.
2. Output Resistance R_o = 0: In this case the output voltage V_o is independent of theoutput current.
3. Open Loop Voltage Gain $\mu = A_V = \pm \infty$

Active filter implementation has three distinct advantages over passiveimplementation.

- First, active filter implementationneed not make use of any inductors: only resistors, capacitors, and activeelements (active elements are restricted to operational amplifiers (op amps) here). Thereason why this is an advantage is that practical inductors (physical approximationsto inductors) tend to be less ideal than are practical capacitors. Practical inductors arecoils of wire, often fine wire, on some sort of core material. The core material haslosses, the wire has resistance, and there is capacitance between the layers of coilwindings. In addition, the coils radiateelectromagnetic energy and can result inunwanted mutual inductance.

- Second, the output resistance of an op amp is very low, especially with thefeedback that is used with common active filter op amp stages. This output resistance(output impedance) is much lower than is the input impedance to the following activefilter op amp stage. Even though the input impedance to a typical active filter op ampstage is frequency dependent and non-resistive this is so. Therefore, the individualactive filter op amp stages, perhaps several being cascaded together, operateindependently of each other (note that this is not true for individual ladder stages ofa passive filter). This allows for the implementation of simple first-order and secondorderactive op amp stages which are then cascaded to yield the overall desiredtransfer function. All that is required is the introduction to a catalog of active filterop amp stages, and no detailed tables are required.

- Third, because of the independence of individual stages, the design andimplementation engineer is not restricted to filters that are in a set of tables. That is, any filter transfer function, with any frequency transformationapplied, can, at least in theory, be implemented with an active op amp filter, withcomplete flexibility as to such parameters as passband ripple, etc.

However, where high power or very high frequencies are involved, activefilter implementation with op amps is prohibited. Therefore, passive filterimplementation is often desired or required.

For simplicity, only ideal op amps will be considered. The op amp symbol isshown on the left side of Figure 5.2, and a more detailed representation of the idealop amp is shown on the right side: the ideal op amp has infinite open-loop gain, zerooutput resistance, and infinite input resistance.

While amplifiers, per se, are not necessary in the implementation of activefilters, they may be desirable for gain adjustment. The inverting amplifier, on the leftside of

Figure 5.3, has the gain$V_o/V_{in} = -R_2/R_1$, The noninverting amplifier, onthe right side of Figure 11.2 has the gain$V_o/V_{in} = 1 + (R_2/R_1)$.

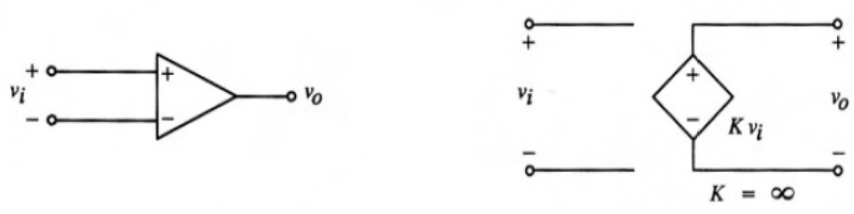


Fig 5.2 : The op amp symbol

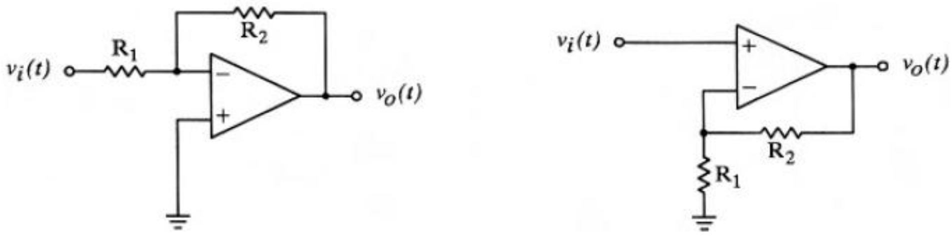


Fig 5.3 : The inverting and noninvertin amplifier.

5.2 First-order stages

In Figure 5.4 is shown a first-order op amp stage that implements **one realpole and no finite zero.** This circuit and corresponding transfer function may be usedwhere a pole on the negative real axis of the s plane is required, such as in odd-orderlowpass filters. It is easy to show that the transfer function for the circuit shown inFigure ….., is as follows:

where the pole is at $S = -1\ /R_2\, C$ and the DC gain for this first-order lowpasstransfer function is $-R_2/R_1$For convenience it is often desirable to normalize atleast one of the component values.

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{R_1\, C}}{S + \frac{1}{R_2\, C}} \qquad [29]$$

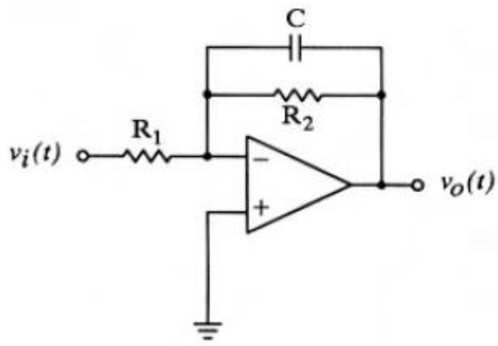


Fig 5.4 : first-order op amp stage that implements one real pole and no finite zero

In Figure 5.5.is shown a first-order op amp stage that implements one realpole and a zero at the origin. This circuit and corresponding transfer function may beused where a pole on the negative real axis of the s plane is required and a zero at theorigin, such as in odd-order highpass filters.

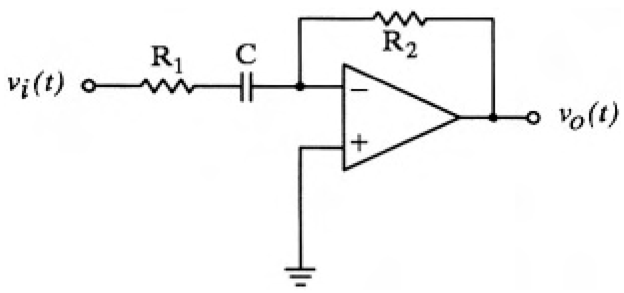


Fig 5.5 : first-order op amp stage that implements one realpole and a zero

The transfer function for the circuit shown in Figure ….is as follows:

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{R_2}{R_1}s}{S+\frac{1}{R_1 C}} \quad [30]$$

where the pole is$S = -1 / R_1 C$ at and the high-frequency gain for this first-order highpass transfer functions$-R_2/R_1$, while cut off frequency $\omega_c = \frac{1}{\sqrt{R_2 C}}$.

5.3 Second-order stages

5.3.1 introduction

The second-order stages presented in this section include stages appropriate forlowpass, highpass, bandpass, bandstop, and all-pass filters.

We first introduce standard second-order transfer functions.The network theoretic background for realization of a frequency-selectivenetwork, such as a filter, using an active device embedded in an array of passiveelements containing only resistances and capacitances is presented next. Ageneral network containing a single-voltage amplifier and passive elements thatcan realize a biquadratic transfer function is then presented.

Sensitivity considerations are introducedand several low-sensitivity second-order filters are presented. Second-order filtersusingOTA (operational transconductance amplifier)are considerednext.

5.3.2 Biquadratic filters or biquads

A second-order transfer function of the form

$$H(s) = \frac{N(s)}{D(s)} = \frac{b_2 S^2 + b_1 S + b_0}{S^2 + a_1 S + a_0} \qquad [31]$$

is called a biquadratic function. Even though, in general, the poles and zeros maylie on the negative real axis, we will assume them to be complex conjugates, sincepoles and zeros on the negative real axis can be realized using passive-RC circuits. In such a case, we may express:

$$H(s) = \frac{N(s)}{D(s)} = H_0 \frac{(s+z)(s+z^*)}{(s+p)(s+p^*)} = H_0 \frac{s^2 + (\omega_z/Q_z)s + \omega_o{}^2}{s^2 + (\omega_p/Q_p)s + \omega_o{}^2} \qquad [32]$$

Biquadratic filters or biquads. These are listed in Table 5.1. It is seen that, exceptfor the OP Amp filter, the zeros are all on the imaginary axis and hence, Qz =∞.

Themeaning of ω_o and Q will become more clear when weconsider a LPF filterof the form:

$$H_{LPF}(s) = \frac{K\omega_o{}^2}{s^2 + (\omega_o/Q)s + \omega_o{}^2} \qquad [33]$$

We can define new parameter which is bandwidth or BW

$$BW = \frac{\omega_o}{Q} \quad [34]$$

Thus, Q and BW are inversely related and hence, the higher the Q, the narrowerthe BW of the filter.

Type of filter	$N(s)$
Low-pass (LP)	$H_o\omega_p^2$
Band-pass (BP)	$H_o(\omega_p/Q_p)s$
High-pass (HP)	$H_o s^2$
All-pass (AP)	$s^2 - (\omega_p/Q_p)s + \omega_p^2$
Notch	$H_o(s^2 + \omega_n^2)$

Note: For all filters,
$\frac{V_2(s)}{V_1(s)} = H(s) = \frac{N(s)}{D(s)}$, with $D(s) = s^2 + (\omega_p/Q_p)s + \omega_p^2$

Table 5.1 : Standard biquadratic transfer functions.

Figure 5.6 showsthe magnitude response for typical biquad LP, HP, BP, and notch filters aswell as the relation between Q and BW.

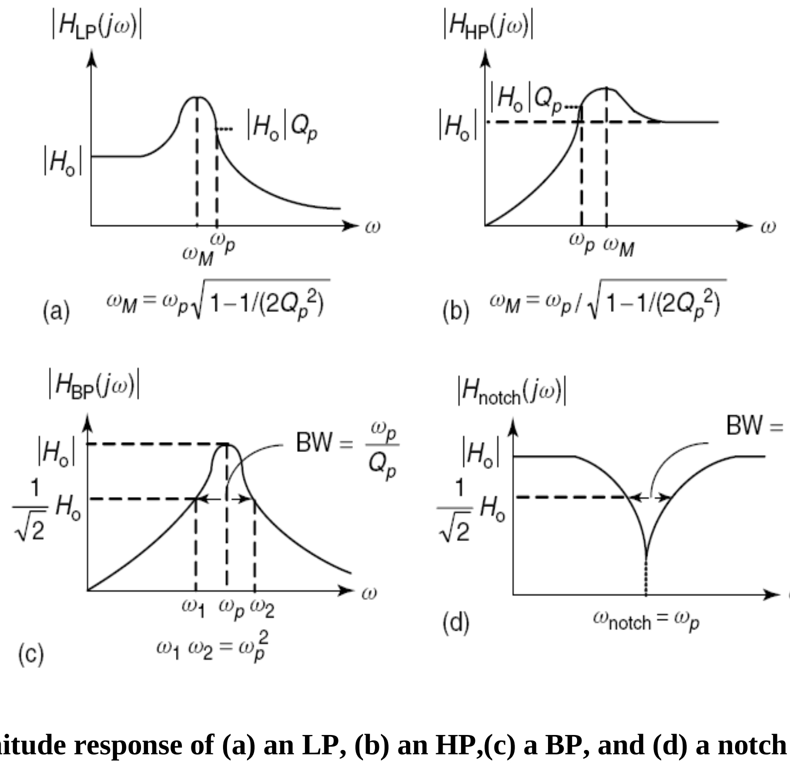


Fig 5.6 : Magnitude response of (a) an LP, (b) an HP,(c) a BP, and (d) a notch filter.

Realization of Single-Amplifier Biquadratic Filters, a passive network consisting of only resistors and capacitorshas all its poles on the negative real axis, and hence cannot have complex poles.Therefore, a passive-RC network cannot give rise to a frequency-selective transferfunction. Imbedding an active device such as a voltage amplifier with a gain Kin a passive-RC network, however, opens up the possibility of realizing a transferfunction with complex poles.

A general configuration for producing a biquadratic transfer function usinga single amplifier of finite gain K, popularly referred to as single-amplifier biquad(SAB) is shown in Figure 5.7.

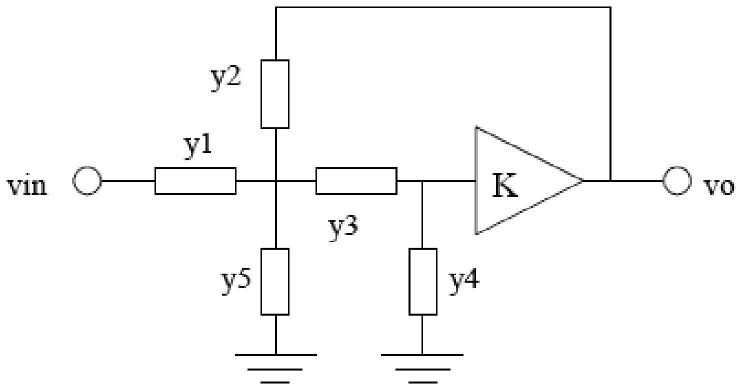


Fig 5.7 : A specific single-amplifier biquad.

Using the method of analysis of constrained networksin conjunction with the nodal suppression technique (see Chapter 2), we can showthat the transfer function V2(s)/V1(s) is given by

$$H(s) = \frac{y_1 y_3 K}{(y_1 + y_2 + y_5)(y_3 + y_4) + y_3(y_4 - y_2 K)} \qquad [35]$$

Choosing the admittances appropriately, it is possible to realize second-order filterswith LP, BP, and HP characteristics as follows:

- ✓ y1 and y3 resistors; y2 and y4 capacitors then H(s) is lowpass.
- ✓ y1 and y4 resistors; y3 and y5 capacitors then H(s) is bandpass.
- ✓ y2 and y4 resistors; y1 and y2 capacitors then H(s) is highpass.

Filters using a single positive gain (K is positive)ideal voltage amplifiers are also known as **S**allen and **K**ey (**SK**) filters (Sallen andKey, 1955)

5.3.3 Sallen and Key Lowpass Circuit

In Figure 5.8 is shown the **Sallen and Key** lowpass circuit, which implementsone complex-conjugate pair of poles with no finite-valued zeros.

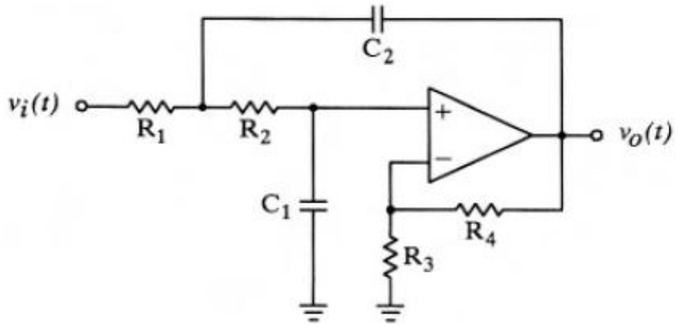


Fig 5.8 : the Sallen and Key lowpass circuit

This circuit may beused to implement one second-order stage in a lowpass filter that has no zeros on$j\omega$ theaxis, such as required with Butterworth, Chebyshev Type I, Bessel, etc. lowpassfilters. The transfer function for the circuit shown in Figure …… is as follows:

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{K/(R_1R_2C_1C_2)}{S^2+\left(\frac{1}{R_1C_2}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}-\frac{K}{R_2C_1}\right)s+\frac{1}{R_1R_2C_1C_2}} \quad [36]$$

where K, the DC gain, is equal to$1 + (R_4/R_3)$ Assuming a complex-conjugate pairof poles it is easy to show that the magnitude of those poles is$1/\sqrt{R_1R_2C_1C_2}$.

$$\left.\begin{aligned} {\omega_o}^2 &= \frac{1}{R_1R_2C_1C_2} \\ H_{\omega=0} &= K \\ BW = \frac{\omega_o}{Q} &= \frac{1}{R_1C_2} + \frac{1}{R_2C_2} + \frac{1-K}{R_2C_1} \end{aligned}\right\} \quad [37]$$

The above equations may be considered as the general design equations. Ina practical case, simplifying assumptions are used to ease the task of design.The rationale behind such simplification lies in the fact that the general designequation contains more parameters (more component values) than are requiredto satisfy the three conditions given by Eqs. 37. Hence, some of thesecomponents can be assigned suitable values. Depending upon the simplifications,several alternative design equations may be arrived at.

Design procedure

To Calculate the value of Sallen Key component, following procedure can be done:

1 – sallen key transfer function can be written as

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{K/(R_1R_2C_1C_2)}{s^2 + \left(\frac{1}{R_1C_2} + \frac{1}{R_2C_2} + \frac{1-K}{R_2C_1}\right)s + \frac{1}{R_1R_2C_1C_2}} \qquad [38]$$

2- Cut off frequency $f_0$can be calculated where $f_0 = \frac{1}{2\pi R_1R_2C_1C_2}$

2- C1 and R3 values can choosen arbitrary.

3- Two new parameters asβ, α where

$$\beta = 2\pi f_0 C_1 \qquad \alpha = \frac{1}{4Q^2} + k - 1 \qquad [39]$$

can be calculated.

4- circuit component can calculated as follows:

$$\left.\begin{aligned} C_2 &= \alpha C_1 \\ R_1 &= \frac{2Q}{\beta} \\ R_2 &= \frac{1}{2Q\alpha\beta} \\ R_4 &= \frac{R_3}{K-1} \end{aligned}\right\} \qquad [40]$$

LPF to HPF transformation

Since we are avoiding inductors, andtrying to realize filters by active-RC networks, this would not be useful. However,if we now perform an impedance transformation (see Chapter 4) on the new HPnetwork, where every impedance z(s) is transformed into another impedance ofvalue (1/s) z(s), then the transfer function of the HP filter will not be affectedprovided the active element is a VCVS or a CCCS; however, a resistor of value R would become a capacitor of (1/R) Farads and an inductor of value $(1/\omega_p{}^2C)$ Henrieswould become a resistor of value $(1/\omega_p{}^2C)$ Ω . Thus, the resulting HP filter wouldthen again be an active-RC filter using the same active elements as those in theLP filter. The combination of the frequency transformation s →ω2p/s followed bythe impedance transformation z(s) → (1/s) z(s) on the resulting network is alsocalled the RC:CR transformation (Mitra, 1967, 1969), and is useful in converting anactive-RC LP filter to an active-RC HP filter; it is noted that this is true only if theactive element is a VCVS (voltage controlled voltage source) such as an OA, or a CCCS (current controlled current source) such as a current conveyoror a current OA. It does not hold good for an RC filter realized using OTAs. TheRC:CR transformation and its effect on an active-RC filter using a VCVSor a CCCSis shown in Table 5.2.

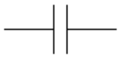
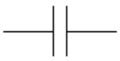
The original transfer function	Transformed transfer function
$H(s)$	$H(\omega_p^2/s)$
R Ω 	$1/R$ F 
C F 	 $1/(\omega_p^2 C)$ Ω
VCVS	VCVS (unaltered)
CCCS	CCCS (unaltered)

Table 5.2: RC:CR transformation and the transformed elements.

5.3.4 Sensitivity

Sensitivity is a measure of the variability of the performance of a filter as a resultof changes in the values of the components and in the characteristics of the activedevice(s) used to implement the filter. Such changes may occur due to aging,anufacturing tolerances, environmental (i.e., temperature and power supply)variations, and so on.

Sensitivity is one of the aspects that can be used to comparethe various filter structures with regard to their robustness toward changes in thecomponent values and the characteristics of the active devices used to implementthe filters. Various researchers have aimed their efforts toward minimizing suchsensitivity and thereby have come up with novel filter structures. In the following,the basic definition of sensitivity will be introducedand illustrated, the use of this definition by considering some known filter structures.

$$s_X{}^Y = \frac{\%\, change\ in\ Y}{\%\, change\ in\ X} = \frac{\Delta Y}{\Delta X} = \frac{dY/Y}{dx/x} = \frac{d(\ln Y)}{d(\ln x)} \qquad [41]$$

If Y is a function of several variables, Y = f (x1, x2, . . . , xn), then the sensitivity of Yw.r.t. xi is given by:

$$s_X{}^Y = \frac{dY/Y}{dx/x} = \frac{d(\ln Y)}{d(\ln x)} \qquad [42]$$

In the above, the symbol ∂ implies the partial derivative. Specifically, a sensitivityof 1/2 means that a 5% change in x would bring a 2.5% change in Y. A sensitivityof −1/2 means that a change of +5% in x would cause a change of −2.5% in Y.

In fig 5.9 and table 5.4, SK filter structures with expressionsfor the ωp and Qp sensitivities is presented. From this table it can be noted that the network in theSK filter has a high sensitivity for Qp.It noticed that; The Sallen-Key is very Q-sensitive to element values, especially for high Q sections.Also bandwidth sensitivity of this

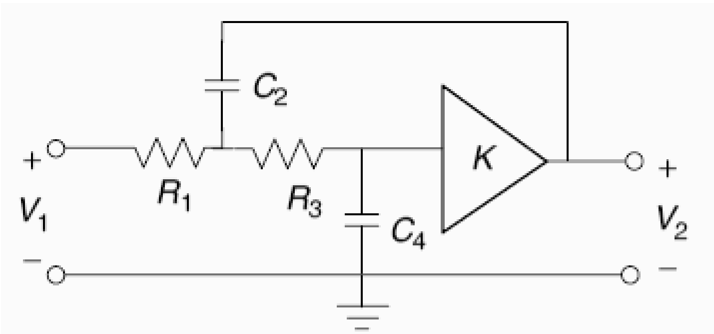


Fig 5.9 : SK filter structures

structure relatedis related with amplification factor K as shown in this equation:

$$S_K^{BW} = -\frac{1}{BW}\frac{K}{R_3C_4}$$

$$-S_{R_3}^{Q_p} = S_{R_1}^{Q_p} = -(1/2) + Q_p\sqrt{\frac{R_3C_4}{R_1C_2}}$$
$$S_{C_2}^{Q_p} = -S_{C_4}^{Q_p} = -(1/2) + Q_p\left(\sqrt{\frac{R_1C_4}{R_3C_2}} + \sqrt{\frac{R_3C_4}{R_1C_2}}\right)$$
$$S_{R_1,R_3,C_2,C_4}^{\omega p} = -(1/2)$$

Table 5.4 :SK filter structures with expressions for the ωp and Qp sensitivities

5.3.4 Design and implement of Sallen Key LPF

Here we will illustrate our design of two order LPF using Sallen Key structure, we used maltisym simulator.

First we calculate the value of the filter component using last procedure, we chose Tl062 op amp then we simulate our design.

Designed Circuit

Here **we will design an LP filter with a pole Q of 4 and a pole frequency of 10^4 rad s^{-1}.**the values of ωp and Qp are given by ωp = 10^4 rad s−1 and Qp = 4, respectively.

Using equations [40] we can calculate the values of circuit components which are:

We choose C1= 25n, R3 = 20 K.

Then we fined the values of the other component which is

C2 = 25 nF.

R2 = 50k.

R1 = 32k.

R4 = 20 K.

Figure 5.10 showssimulated circuit using Multisim simulator.

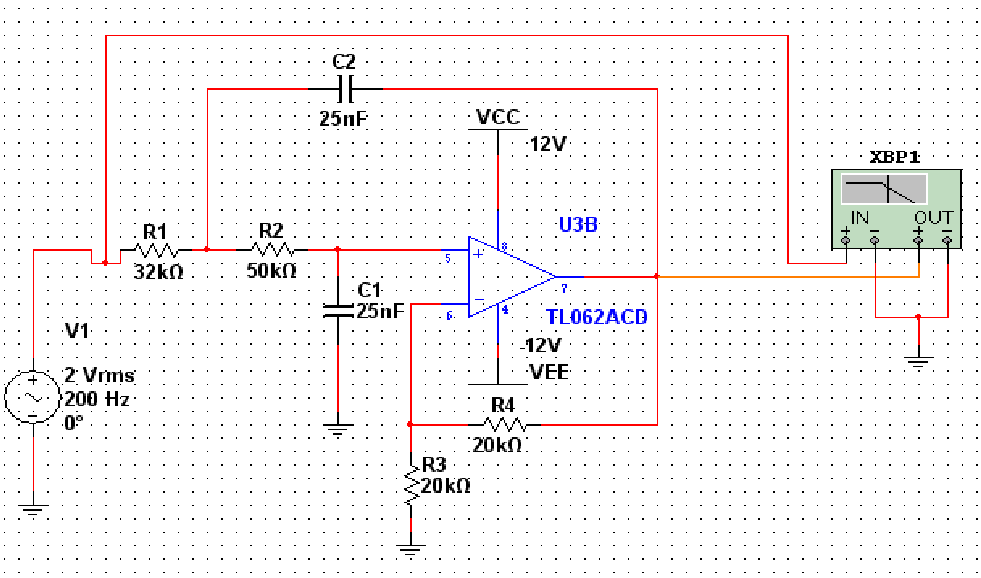


Fig 5.10 : Simulated filter structures

Simulation results shows in figure 5.11, we denoted to the cut off frequency which equal to 173 Hz. It noticed that the amplitued level of the frequency response equal to about 3dB, which denote that; the active filter has a gain, while passive one has'nt.

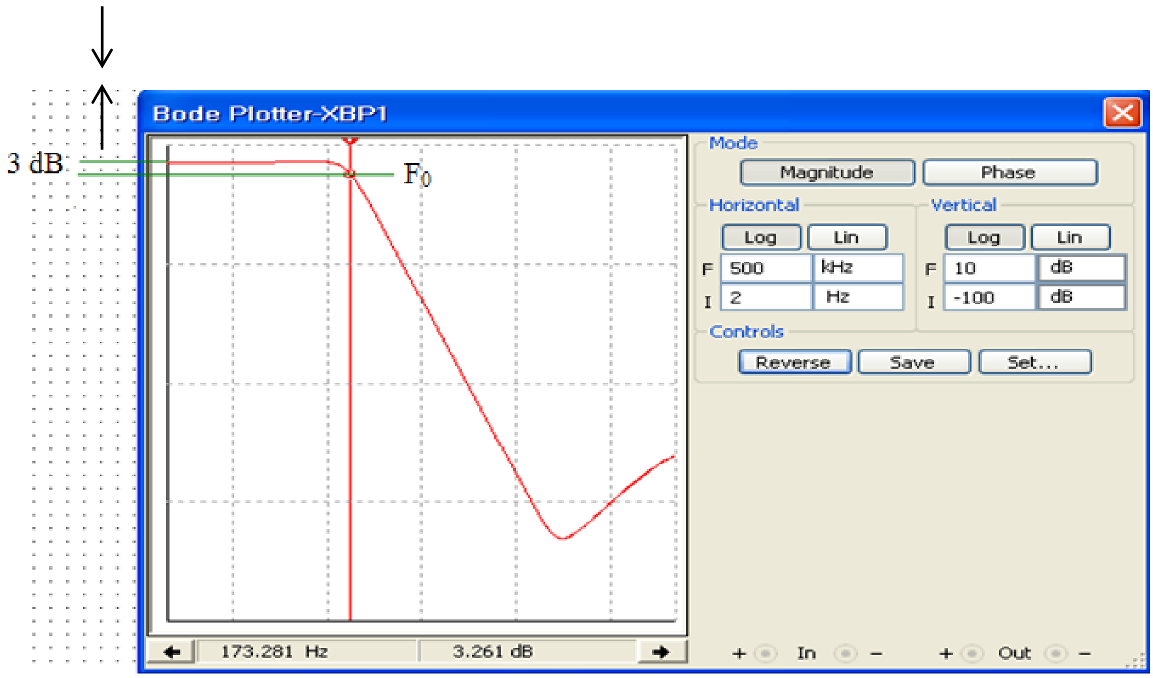


Fig 5.11 :Simulation results

Then we studied the effect of the value reverse resistore R4 on the quality factor "Q", figure 5.12 shows the simulation results, which shows that; the quality factor increased by increasing R4 value.

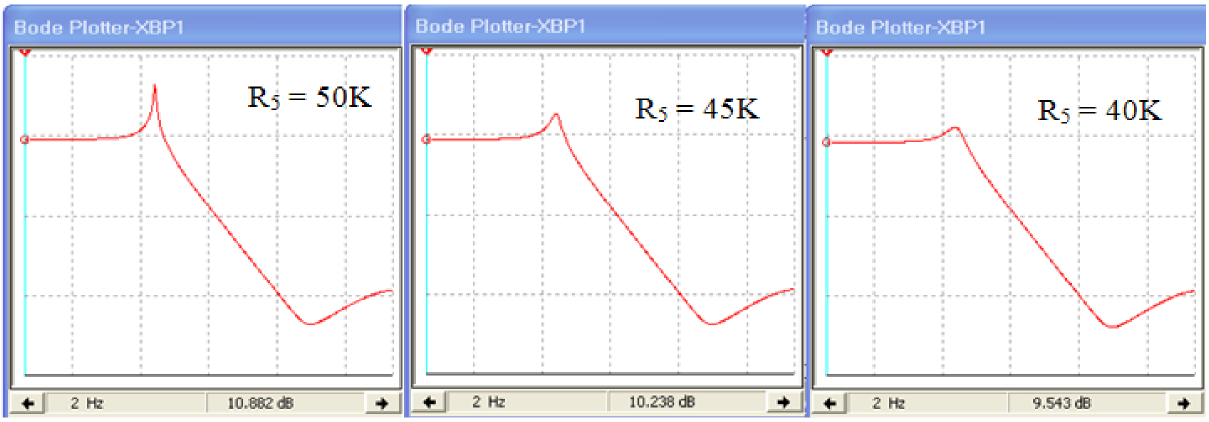


Fig 5.12 : Simulation results of changing the value of R4

We implimented the designed circuit on terst board as shown in figure 5.13

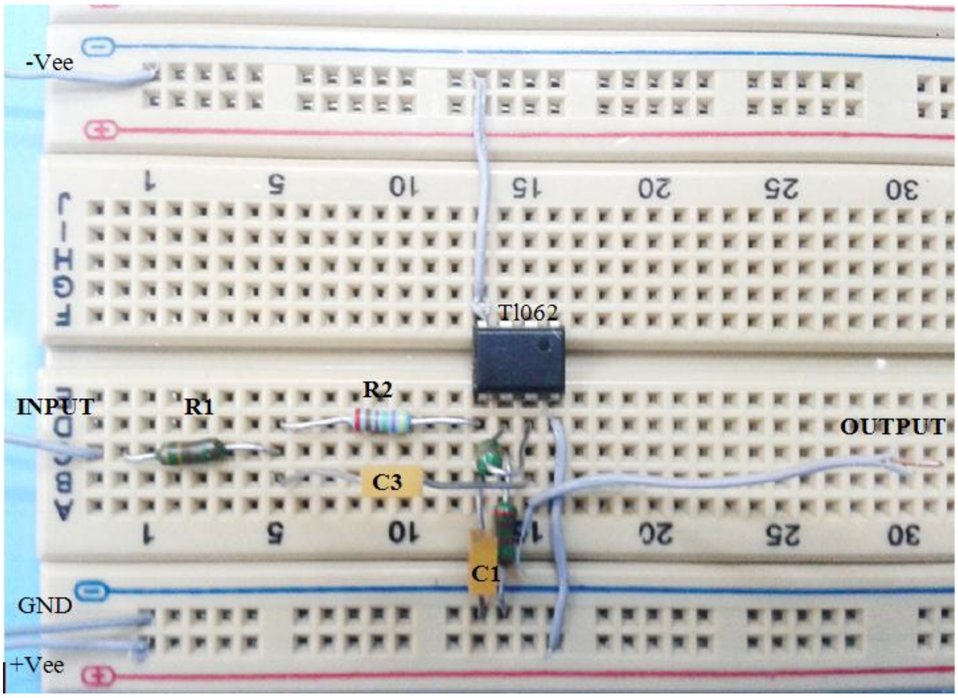


Fig 5.13 : implimented circuit.

5.4Higher-Order Active Filters

the particular choice of realizing a higher-orderfilter is based on a number of factors such as sensitivity, simplicity of design,power dissipation, simplicity of tuning, dynamic range, noise, implementation inintegrated form, and economic considerations.

Component Simulation Technique

It is common knowledge that filters with good frequency selectivity have to be oforder higher than 2. Classically, LC ladder filters were used to accomplish this. It hasbeen shown that a doubly terminated lossless LC ladder structure has minimumsensitivity to component variations in the frequency band of interest (Orchard,1966). Thus, the performance of LC ladder filters is very reliable and stable. Withthe advent of electronic filters, it has become a common practice to replicate theoperation of an LC ladder by means of active filter components to preserve thesame low-sensitivity feature. One approach is to implement the operation of aninductance using active-RC components, and then replace each L in the LC ladderby the simulated inductance and expect that the overall filter will behave in thesame manner as the prototype LC filter. This method is very useful in the caseof HP filters, where the inductance is grounded; however, in the case of a filterlike the LP filter, where the inductors are floating, this is not very suitable as it isdifficult to simulate accurately a floating inductor. In such a case, a transformationis introduced to convert the inductors into resistors; but, in doing so, the capacitorsget converted to ‘‘supercapacitors’’ or ‘‘**f**requency-**d**ependent **n**egative **r**esistances(FDNRs),’’ which are then replaced by active-RC circuits (Bruton, 1969, 1980). **This approach of replacing inductors or FDNRs by active-RC circuits is known as the component simulation technique.**

5.4.2 FDNR or Super-Capacitor in Higher-Order Filter Realization

LP ladder filters have inductances in the seriesbranches. Thus, the active-RC implementation would require simulating floatinginductances. Anattractive alternative in this case stems from applying the impedance transformationto the elements of the LP LC ladder filter. Thus, if theimpedances of the various elements of a doubly terminated LC ladder are multipliedby a factor 1/ks, that is,

$$Z_i(s) \rightarrow \frac{1}{Ks} Z_i(s) \quad [43]$$

then a resistor of value R is transformed to a capacitor of value k/R, an inductor ofvalue L becomes a resistor of value L/k, and a capacitor of valueC_{is} transformed intoan element whose impedance is $1/s^2kC = 1/s^2D$, which corresponds to an FDNRof value −1/ω2kC. This transformation is also known as Bruton's transformation, such an impedancetransformation does not affect a TF (transfer function) of the filter. Hence, for any LP filtersubjected to such a transformation, the TF will remain unaltered. Thus,if an LP LC ladder filter is transformed by such a transformation, we will have astructure consisting of only resistors, capacitors, and FDNRs.

An example of anLP LC ladder and the corresponding structure after the transformation are shownin Figures 5.10a and 5.10b, where D1 and D2 correspond to FDNRs of values $kC_1$and kC_2. This structure can be realized as an active-RC filter provided we can implement theFDNR element using active devices and RC elements.

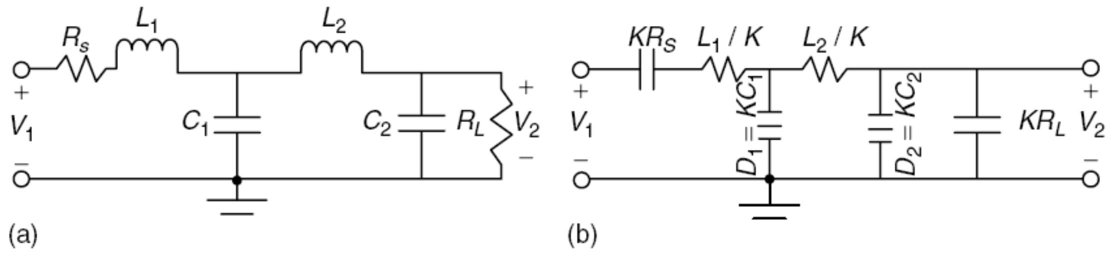


Fig 5.14: (a) A doubly terminated LC LP filter and
(b) the same filter after the impedance transformation

Antoniou's GIC described earlier can readily be used to implement the FDNR, as an example we can consider the GIC of Figure 5.11. If $Z_1 = Z_L = 1/sCx$, and Z2 = Z3 = R andZ4 = Rx, then, from Eq. (43), we see that the DPI at port 1 is given by

$$Z_{in} = \frac{1}{S^2 {C_x}^2 R_x} = \frac{1}{S^2 D} [44]$$

where D, the value of the FDNR (or the 1/s2 element), is given by ${C_x}^2 R_x$. We couldalso have chosen $Z_1 = Z_3 = 1/sC_x$, $Z_2 = Z_L = R$ and $Z_4 = R_x$. This would alsogivethe input impedance at port 1 to be $Z_{in} = \frac{1}{S^2 {C_x}^2 R_x} = \frac{1}{S^2 D}$. It can be noticed that,in view of the impedance transformation Eq. (44), the terminal resistances areconverted to capacitances, thereby breaking the DC path, which, however, mustexist in an LP filter.

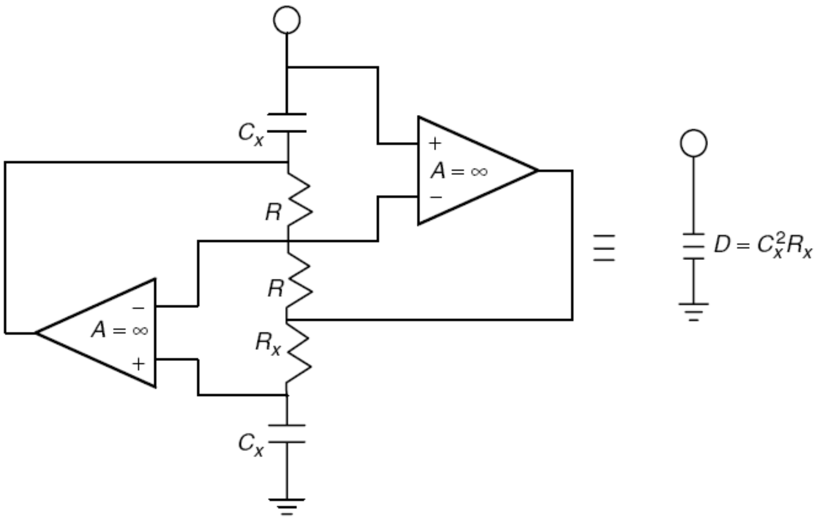


Fig 5.15A typical FDNR subcircuit to be used in the realization of the LP filter of Figure 5.10

To overcome this problem, the terminal capacitances areshunted by very high resistances. The values of these resistances are chosen suchthat the DC gain of the filter using FDNRs is maintained as closely as possibleto that in the prototype ladder filter (Figure 7.10a). It should be observed that theFDNR realized above with the GIC is a grounded one and not a floating one. It maybe mentioned in passing that it is possible to obtain a floating FDNR using twoGICs.

5.4.2 Sensitivity Considerations

Like the simulated inductor, the FDNR retains the excellent sensitivity properties ofthe LP prototype. This is true not only w.r.t. the transformed prototype resistors andcapacitors that appear directly in the active realization, but also w.r.t. the elementsof the GIC that realize the FDNRs. This can be appreciated from the expression forthe super-capacitor D = $C_1 C_L R_2 R_4 / R_3$. Obviously, the sensitivity of D with respect to the elements of the FDNR is either +1 or −1. Thus, the sensitivity of any networkfunction with respect to the elements of the FDNR will be ±1 times the sensitivityof the same network function to the prototype capacitor that has been transformedinto the super-capacitor.

Chapter 6
Conclusion

In this work we are consider basic concepts on filter design and analysis applicable to most filtertypes. We have declared that; passive filters contain no active elements in the filter implementation. It is found that; passive filter implementation may have many forms; in this thesis only ladder implementations are considered.

Main filter's characteristics have been reviewed, as selectivity and stability. Filter parameters also have been reviewed; we found that these characteristics must be studied when the filter is designed. Choosing filter degree is necessary to determine filter selectivity, while choosing number of zeros and poles of the filter and their location in S plane determine the stability of the filter.

Filter parameters also explained, these parameters determine frequency response characteristics which are cut-off frequency, stopband frequency, passsband, stopband, transition band, passband attenuation and stopband attenuation.

Also main classes of filters have been illustrated; we reviewed the main types related with frequency behavior, which are LPFs, HPFs, and BPFs. Other main types related with frequency response which are Butterworth and chebyshev.

Before starting filter design, we determined the filter's rule in frequency domain; in another word we chose their type from frequency domain overview, which is LPF type.

Since passive filters are usually designed starting with a magnitude response approximation, so specific filter response type is one of the main steps of filter design.

Maximally-flat magnitude is very commonly used in practice. It is designed to yield a maximally-flat magnitude response in the passband (actually at DC) and is frequently referred to as the maximally-flat design. The design is based on Butterworth polynomials.

Other types have ripple in thepassband of an equal across-the-band magnitude and of a specified amount; they called Chebyshev Type, also their response monotonically falling off through the transition band and the stopband.These filters will usually meet a set of magnitude specifications with a lower orderthan will a comparable Butterworth design.

One of famous methods of filters design have been explained, this method uses tables give the normalized values of low pass filter components, the tables have several kinds, which related with the filter response shapes, designer can choose suitable table depending on filter characteristics as response shape, and pass band ripple, then choose normalized LPF component values depending on filter degree, then he can calculate the denormalized values of LPF prototype depending on cut off frequency value.

Transformation rules can be applied to the elements values of the lowpass prototype implementation when designer want to design BPF or HPF.

Active filters design rules have been explained, First active filters implementation advantages over passive implementation has been illistrated.

First order, second order design rules have been studied. Sallen key filters which present second order active filters have been explained, one of its design prossedure has been studied. Then transfer rules from LP to HP filters have been illustrated.

In this study it is found that; active filters are preffred when quality factor of the filter is the main gole in our design. Also when the big size of the coils is aproblem, the active filters are preffered.

When the filter is designed to work at very high frequencies, or high powers, active elements can't be used; so passive must be used.

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