Real-Time Systems

RMS, EDF Schedulers (contd) -- Exact Analysis

Exact Analysis (necessary & sufficient)

Critical Zone Theorem:

For a set of independent periodic tasks, if a task T_i meets its first deadline $d_i <= p_i$, when all other higher priority tasks are started (ie., ready) at the same time, then it meets all its future deadlines with any other task start times.

Completion Time Test

Let there be n tasks ordered in decreasing priority. Consider any task T_i. The workload over [0,t] (for arbitrary t > 0) due to all tasks of equal or higher priority than Ti is given by

$$W_i(t) = \sum_{j=1}^{i} c_j \left\lceil \frac{t}{p_j} \right\rceil$$

The term $\left\lceil \frac{t}{p_j} \right\rceil$ represents the number of times task T_j arrives in time t, and therefore $c_j \left\lceil \frac{t}{p_j} \right\rceil$ represents its computational demand in time t.

Completion Time Test (Contd.)

Suppose that task Ti completes its execution exactly at time t. This means that the total cumulative demand from the i tasks up to time t, $W_i(t)$, is exactly equal to t, that is, $W_i(t) = t$.

A method for finding the completion time of task T_i, that is, the time at which $W_i(t) = t$, is known as completion time test.

Completion Time Test (Contd.)

$$Set \quad t_0 = \sum_{j=1}^{i} c_j$$

$$t_1 = W_i(t_0)$$

$$t_2 = W_i(t_1)$$

$$t_3 = W_i(t_2)$$

$$\vdots$$

$$t_k = W_i(t_{k-1})$$

$$Stop \quad when \quad W_i(t_k) = t_k$$

A task T_i is schedulable if Wi <= di, where $W_i(t)=t$. An entire task set is schedulable if this condition holds for all the tasks in the set.

Completion Time Test — Example

Task T_1: $c_1 = 20$; $p_1 = 100$; $d_1 = 100$.

Task T_2: $c_2 = 30$; $p_2 = 145$; $d_2 = 145$.

Task T_3: $c_3 = 68$; $p_3 = 150$; $d_2 = 150$.

This task set fails the (utilization-based) schedulability test for RMS. So, perform completion time test for T1, T2, T3. Task T3′s completion time test is as follows.

$t_0 = c_1 + c_2 + c_3 = 20 + 68 + 30 = 118.$

$t_1 = W_3(t_0) = 2c_1 + c_2 + c_3 = 40 + 68 + 30 = 138.$

$W_3(t_1) = 2c_1 + c_2 + c_3 = 40 + 68 + 30 = 138 = t_1.$

Task T_3 is schedulable!; Tasks T1 and T2 too.

Deadline monotonic scheduling (DMS)

- Task Ti: (ci, pi, di); with relative deadline di <= pi
- Assigns priority based on di; smaller the di, higher the priority

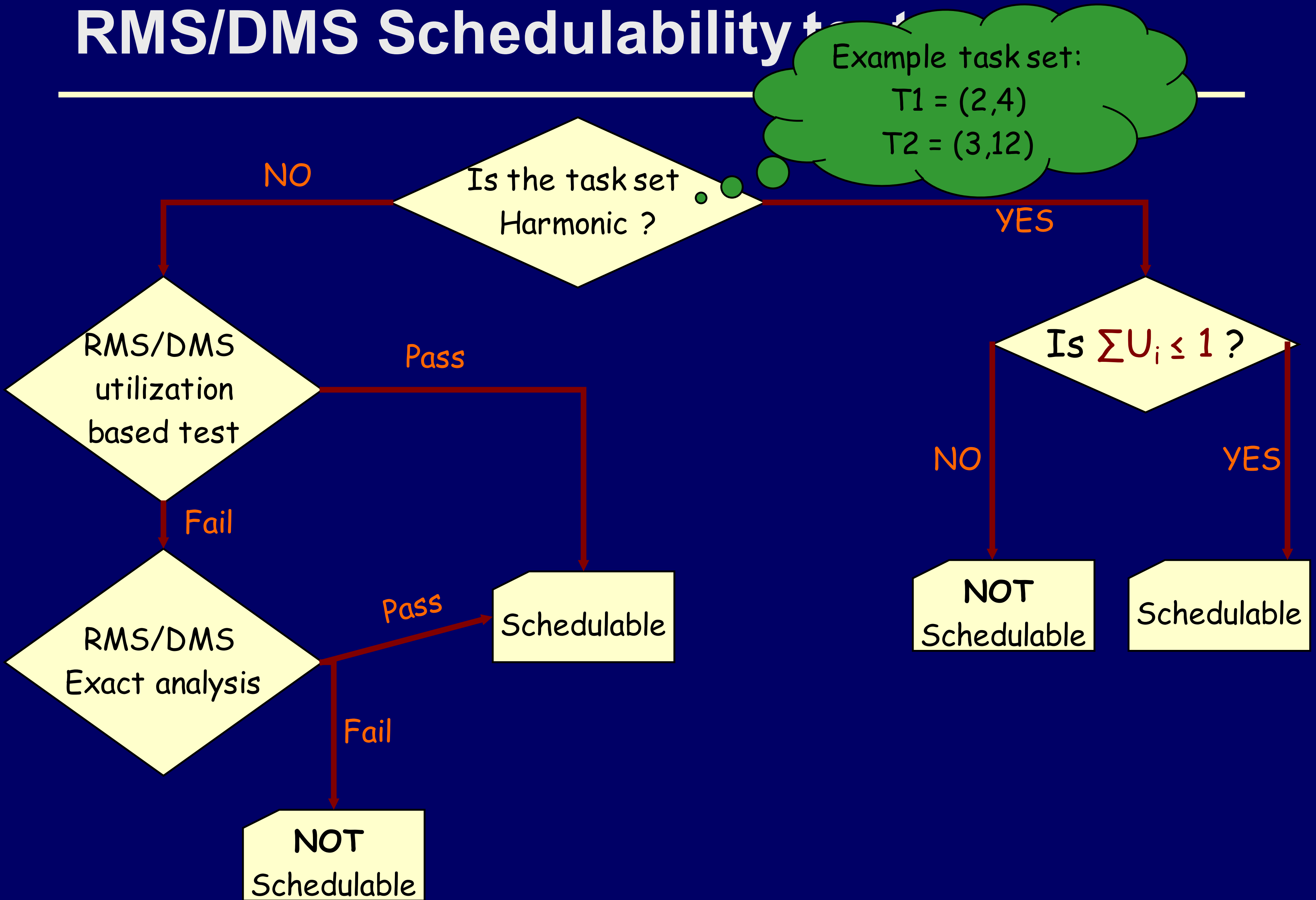
- Similar to RMS utilization-based schedulability test, except ci/di used instead of ci/pi

$$\sum C_i/d_i \leq n(2^{1/n} - 1)$$

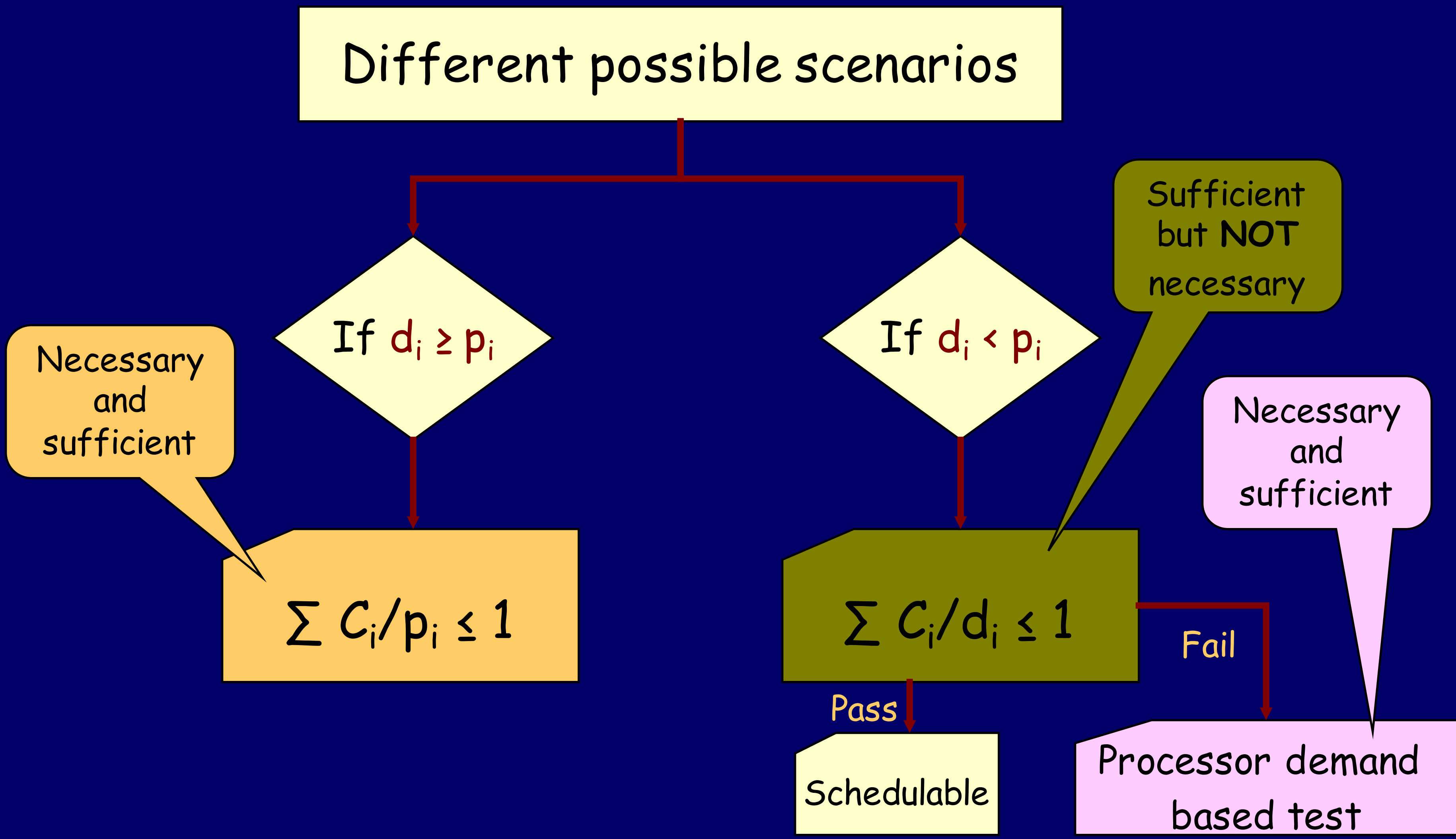
- Similar to RMS exact analysis, except the ordering of tasks is based on di instead of pi

- Example: (ci,pi,di): (3,20,7), (2,5,4), (2,10,9). This task set is schedulable even though Sum(ci/di) > 1.
- DMS is also an optimal fixed-priority scheduling algorithm; it is a generalization of RMS.

RMS/DMS Schedulability t



EDF Revisited: Schedulability test



Periodic task scheduling - summary

	Di = Pi	Di ≤ Pi
Static priority	RMS Processor Utilization test	DMS Exact Analysis
Dynamic priority	EDF/LLF Processor utilization test (U ≤ 1)	EDF Processor demand based test (not covered)