Three Dimensional Viewing

Three Dimensional Viewing Pipeline

Procedures for generating a computer-graphics view of a three-dimensional scene are somewhat analogous to the processes involved in taking a photograph. First of all, we need to choose a viewing position corresponding to where we would place a camera. We choose the viewing position according to whether we want to display a front, back, side, top, or bottom view of the scene. We could also

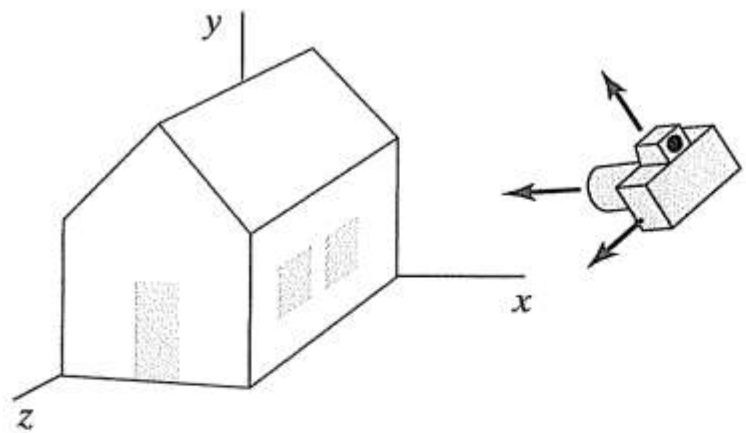


FIGURE 7-10 Photographing a scene involves selection of the camera position and orientation.

pick a position in the middle of a group of objects or even inside a single object, such as a building or a molecule. Then we must decide on the camera orientation (Fig. 7-10). Which way do we want to point the camera from the viewing position, and how should we rotate it around the line of sight to set the "up" direction for the picture? Finally, when we snap the shutter, the scene is cropped to the size of a selected clipping window, which corresponds to the aperture or lens type of a camera, and light from the visible surfaces is projected onto the camera film.

Figure 7-11 shows the general processing steps for creating and transforming a three-dimensional scene to device coordinates. Once the scene has been modeled in world coordinates, a viewing-coordinate system is selected and the description of the scene is converted to viewing coordinates. The viewing coordinate system defines the viewing parameters, including the position and orientation of the projection plane (view plane), which we can think of as the camera film plane. A two-dimensional clipping window, corresponding to a selected camera lens, is defined on the projection plane, and a three-dimensional clipping region is established. This clipping region is called the **view volume,** and its shape and size depends on the dimensions of the clipping window, the type of projection we choose, and the selected limiting positions along the viewing direction. Projection operations are performed to convert the viewing-coordinate description of the scene to coordinate positions on the projection plane. Objects are mapped to normalized coordinates, and all parts of the scene outside the view volume are clipped off. The clipping operations can be applied after all device-independent coordinate transformations (from world coordinates to normalized coordinates)

are completed. In this way, the coordinate transformations can be concatenated for maximum efficiency.

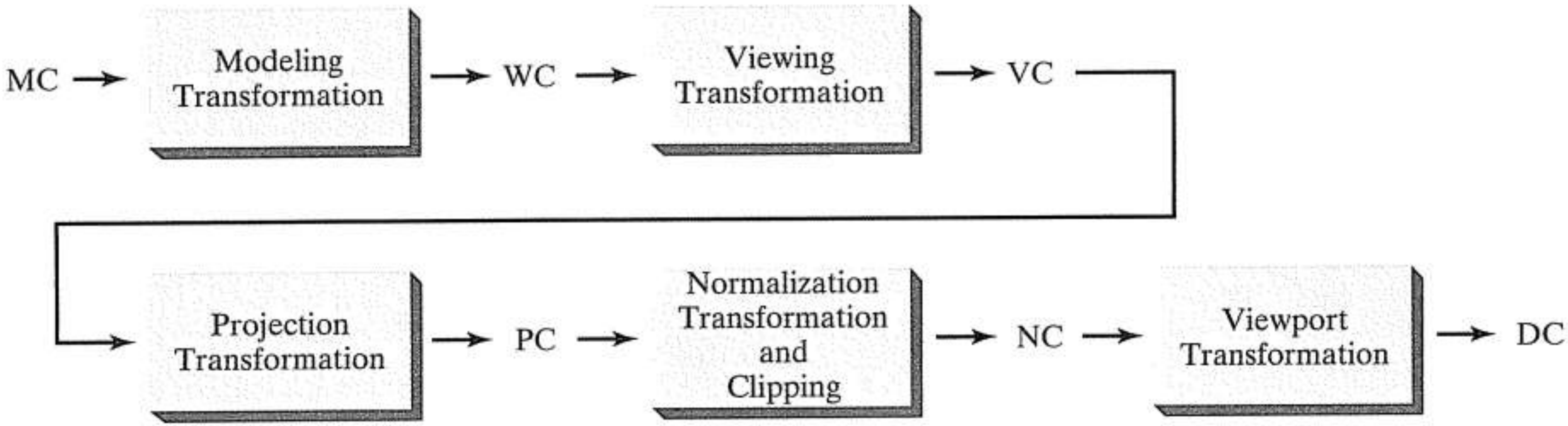
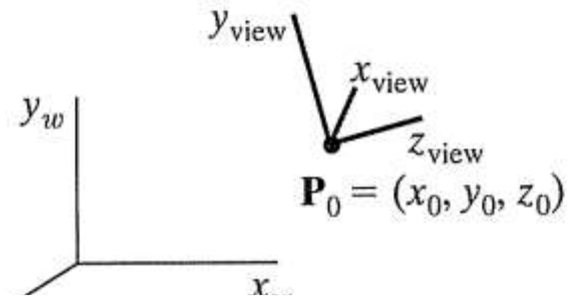


FIGURE 7-11 General three-dimensional transformation pipeline, from modeling coordinates to world coordinates to viewing coordinates to projection coordinates to normalized coordinates and, ultimately, to device coordinates.

Three Dimensional Viewing-coordinate Parameters

P_0:view point, Eye position or camera position.

Z view: View Plane normal vector N



7-3 THREE-DIMENSIONAL VIEWING-COORDINATE PARAMETERS

A right-handed viewing-coordinate system, with axes x_{view}, y_{view}, and z_{view}, relative to a right-handed world-coordinate frame.

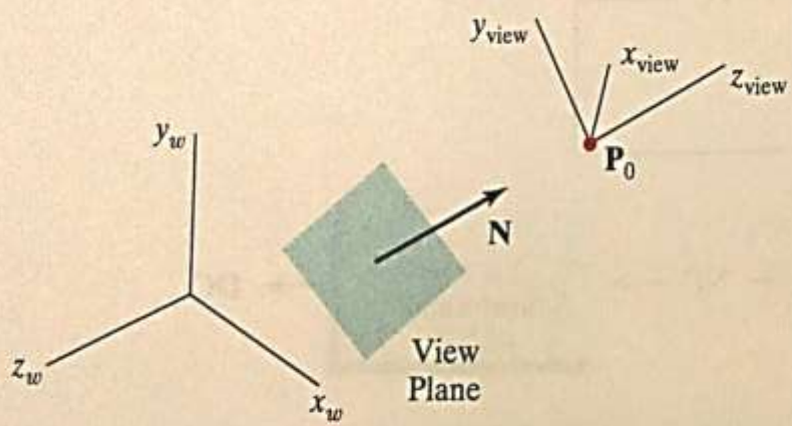


FIGURE 7-13 Orientation of the view plane and view-plane normal vector **N**.

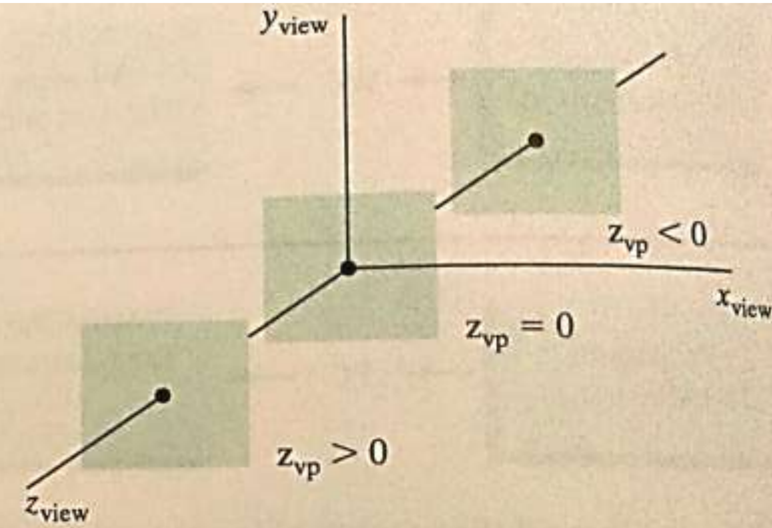
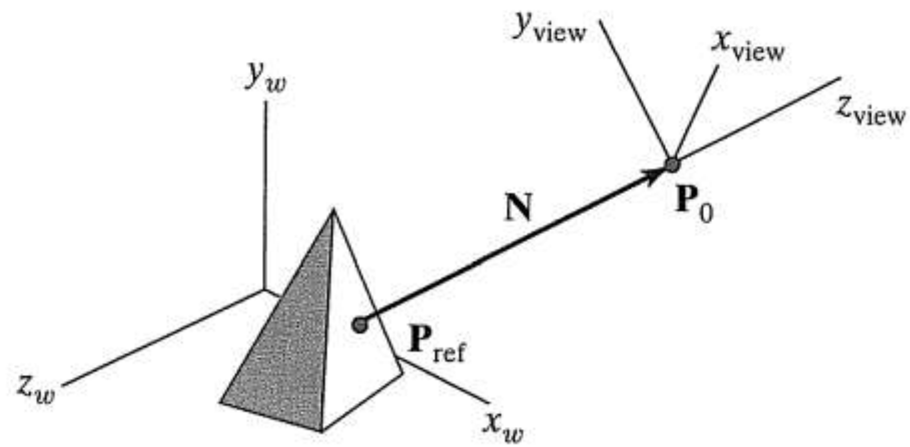


FIGURE 7-14 Three possible positions for the view plane along the z_{view} axis.

FIGURE 7-15 Specifying the view-plane normal vector **N** as the direction from a selected reference point $\mathbf{P}_{ref}$ to the viewing-coordinate origin $\mathbf{P}_0$.



nonoverlapping, contiguous solids (usually cubes).

Transformation from world to viewing coordinates

In the three-dimensional viewing pipeline, the first step after a scene has been constructed is to transfer object descriptions to the viewing-coordinate reference frame. This conversion of object descriptions is equivalent to a sequence of transformations that superimposes the viewing reference frame onto the world frame. We can accomplish this conversion using the methods for transforming between coordinate systems described in Section 5-15:

(1) Translate the viewing-coordinate origin to the origin of the world-coordinate system.

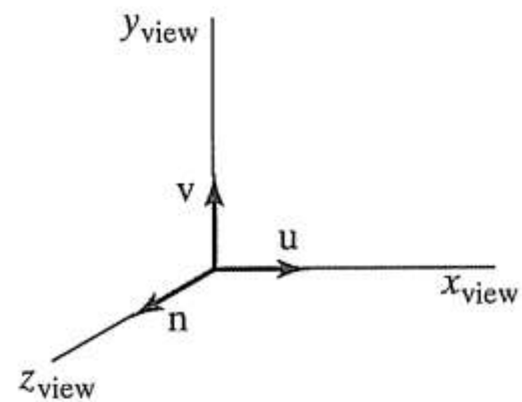
(2) Apply rotations to align the x_{view}, y_{view}, and z_{view} axes with the world x_w, y_w, and z_w axes, respectively.

The viewing-coordinate origin is at world position $\mathbf{P} = (x_0, y_0, z_0)$. Therefore, the matrix for translating the viewing origin to the world origin is

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (7\text{-}2)$$

For the rotation transformation, we can use the unit vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{n}$ to form the composite rotation matrix that superimposes the viewing axes onto the world frame. This transformation matrix is

$$\mathbf{R} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



where the elements of matrix $\mathbf{R}$ are the components of the $\mathbf{uvn}$ axis vectors.

The coordinate transformation matrix is then obtained as the product of the preceding translation and rotation matrices:

$$\mathbf{M}_{WC,\,VC} = \mathbf{R} \cdot \mathbf{T}$$

Projection Transformations

In the next phase of the three-dimensional viewing pipeline, after the transformation to viewing coordinates, object descriptions are projected to the view plane. Graphics packages generally support both parallel and perspective projections.

In a **parallel projection,** coordinate positions are transferred to the view plane along parallel lines. Figure 7-22 illustrates a parallel projection for a straight-line segment defined with endpoint coordinates $\mathbf{P}_1$ and $\mathbf{P}_2$. A parallel projection preserves relative proportions of objects, and this is the method used in computer-aided drafting and design to produce scale drawings of three-dimensional objects. All parallel lines in a scene are displayed as parallel when viewed with a parallel projection. There are two general methods for obtaining a parallel-projection view of an object: We can project along lines that are perpendicular to the view plane, or we can project at an oblique angle to the view plane.

For a **perspective projection,** object positions are transformed to projection coordinates along lines that converge to a point behind the view plane. An example of a perspective projection for a straight-line segment, defined with endpoint coordinates $\mathbf{P}_1$ and $\mathbf{P}_2$, is given in Fig. 7-23. Unlike a parallel projection, a perspective projection does not preserve relative proportions of objects. But perspective views of a scene are more realistic because distant objects in the projected display are reduced in size.

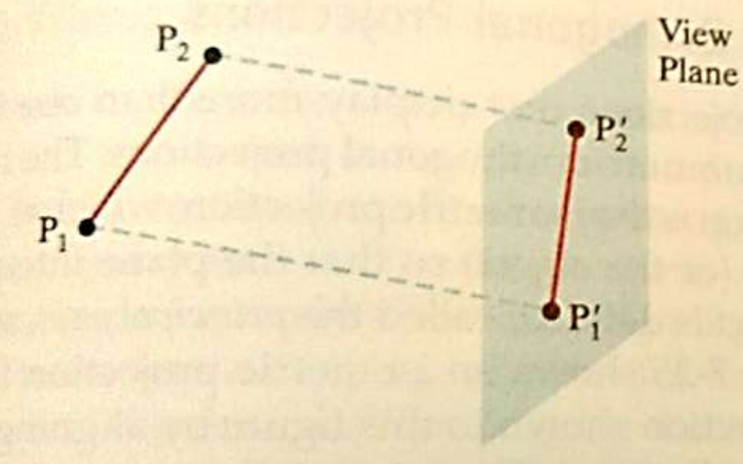


FIGURE 7-22 Parallel projection of a line segment onto a view plane.

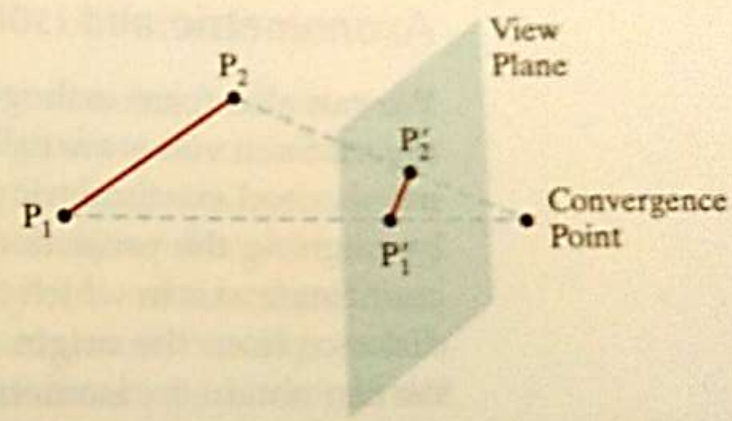
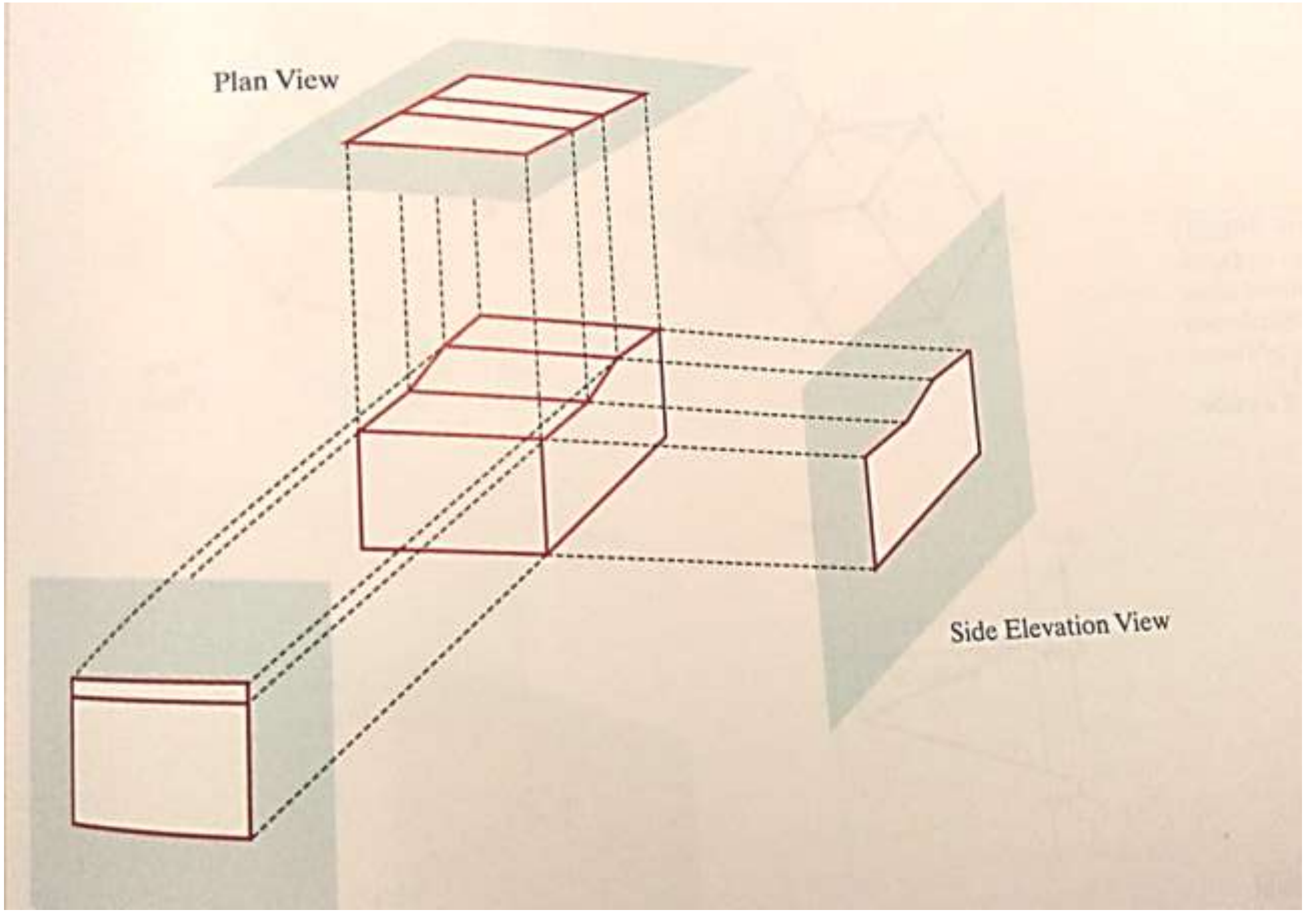


FIGURE 7-23 Perspective projection of a line segment onto a view plane.

ORTHOGNAL PROJECTIONS

A transformation of object descriptions to a view plane along lines that are all parallel to the view-plane normal vector **N** is called an **orthogonal projection** (or, equivalently, an **orthographic projection**). This produces a parallel-projection transformation in which the projection lines are perpendicular to the view plane. Orthogonal projections are most often used to produce the front, side, and top views of an object, as shown in Fig. 7-24. Front, side, and rear orthogonal projections of an object are called *elevations;* and a top orthogonal projection is called a *plan view*. Engineering and architectural drawings commonly employ these orthographic projections, since lengths and angles are accurately depicted and can be measured from the drawings.



Front Elevation View

FIGURE 7-24 Orthogonal projections of an object, displaying plan and elevation views.

Axonometric and Isometric Orthogonal Projections

We can also form orthogonal projections that display more than one face of an object. Such views are called **axonometric** orthogonal projections. The most commonly used axonometric projection is the **isometric** projection, which is generated by aligning the projection plane (or the object) so that the plane intersects each coordinate axis in which the object is defined, called the *principal axes*, at the same distance from the origin. Figure 7-25 shows an isometric projection for a cube. We can obtain the isometric projection shown in this figure by aligning the view-plane normal vector along a cube diagonal. There are eight positions, one in each octant, for obtaining an isometric view. All three principal axes are foreshortened equally in an isometric projection, so that relative proportions are maintained. This is not the case in a general axonometric projection, where scaling factors may be different for the three principal directions.

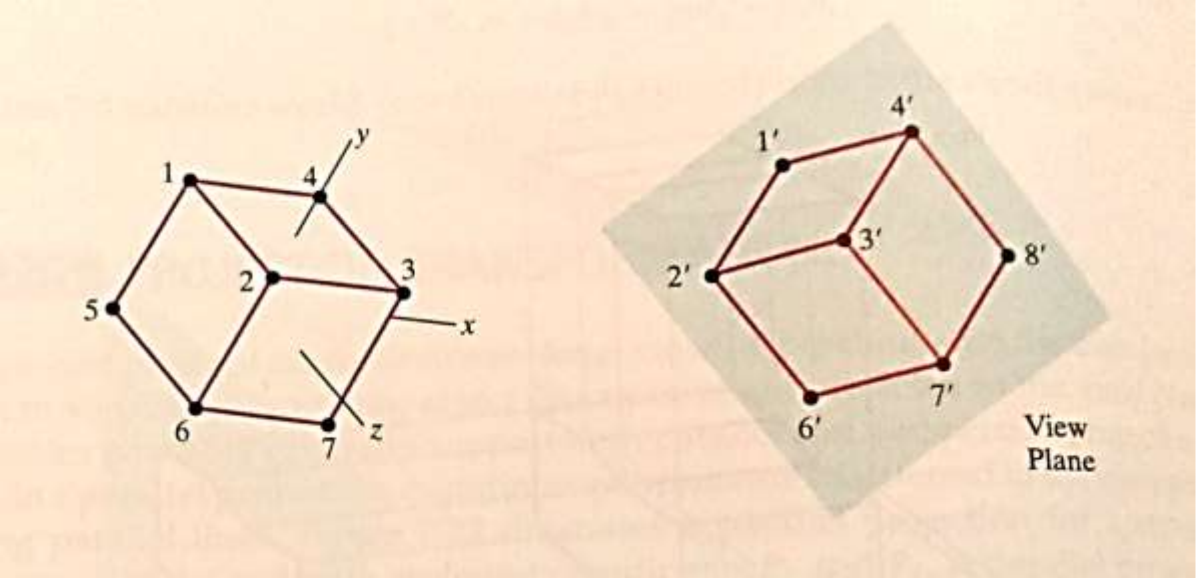


FIGURE 7-25 An isometric projection of a cube.

Orthogonal Projection Coordinates

With the projection direction parallel to the z_{view} axis, the transformation equations for an orthogonal projection are trivial. For any position (x, y, z) in viewing coordinates, as in Fig. 7-26, the projection coordinates are

$$x_p = x, \qquad y_p = y \tag{7-6}$$

The z-coordinate value for any projection transformation is preserved for use in the visibility determination procedures. And each three-dimensional coordinate point in a scene is converted to a position in normalized space.

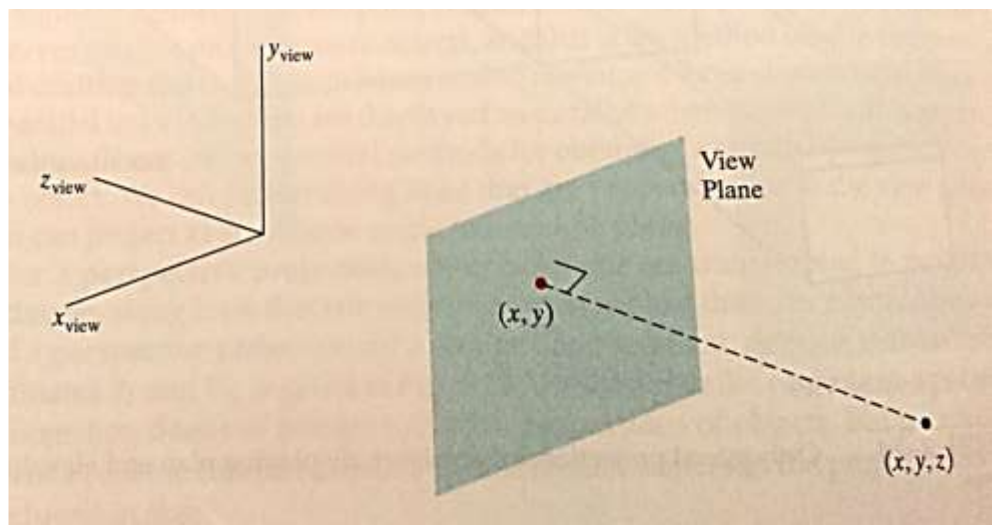


FIGURE 7-26 Orthogonal projection of a spatial position onto a view plane.

Clipping Window and Orthogonal-Projection View Volume

The edges of the clipping window specify the x and y limits for the part of the scene that we want to display. These limits are used to form the top, bottom, and two sides of a clipping region called the **orthogonal-projection view volume.** Since projection lines are perpendicular to the view plane, these four boundaries are planes that are also perpendicular to the view plane and that pass through the edges of the clipping window to form an infinite clipping region, as in Fig. 7-28.

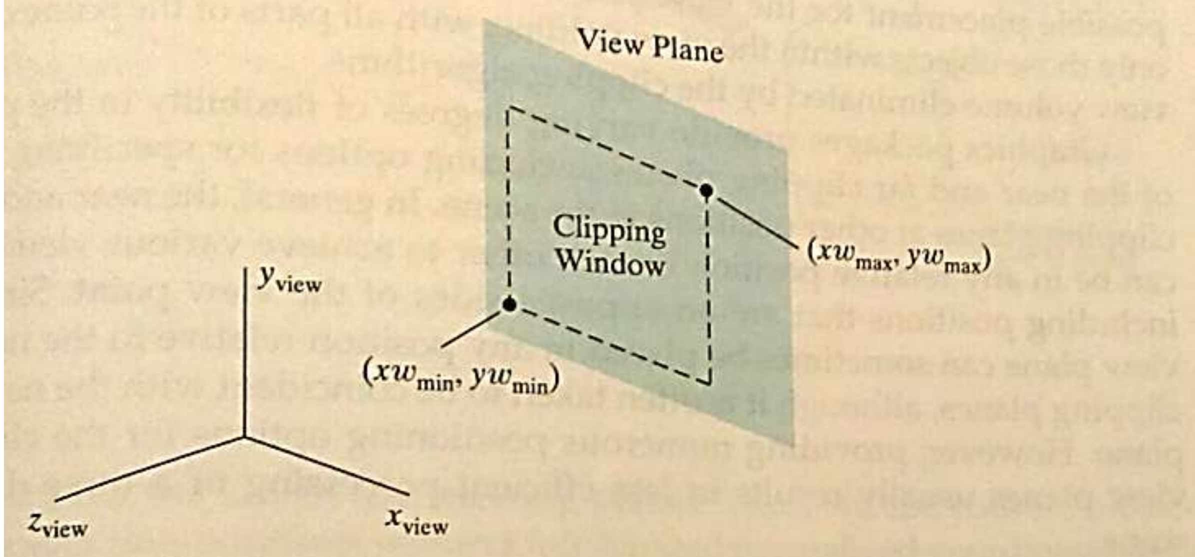


FIGURE 7-27 A clipping window on the view plane, with minimum and maximum coordinates given in the viewing reference system.

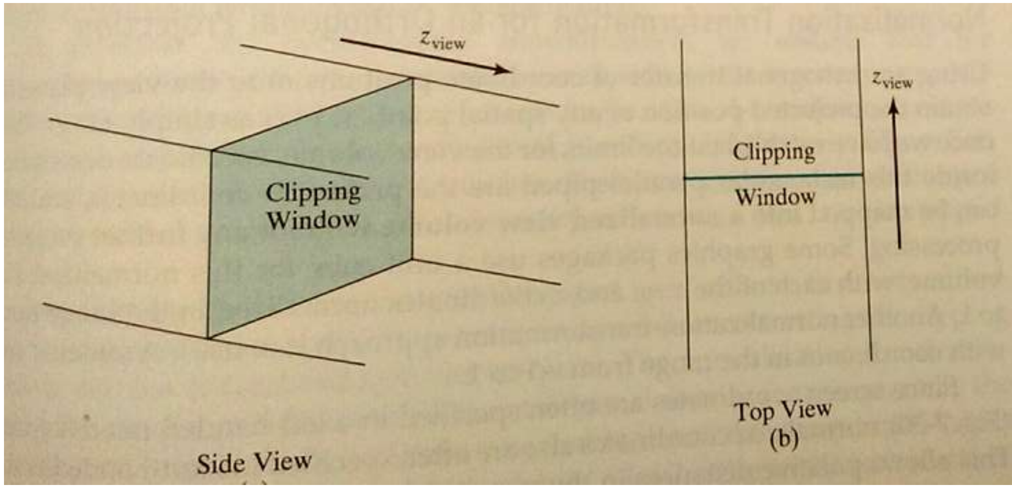


FIGURE 7-28 Infinite orthogonal-projection view volume.

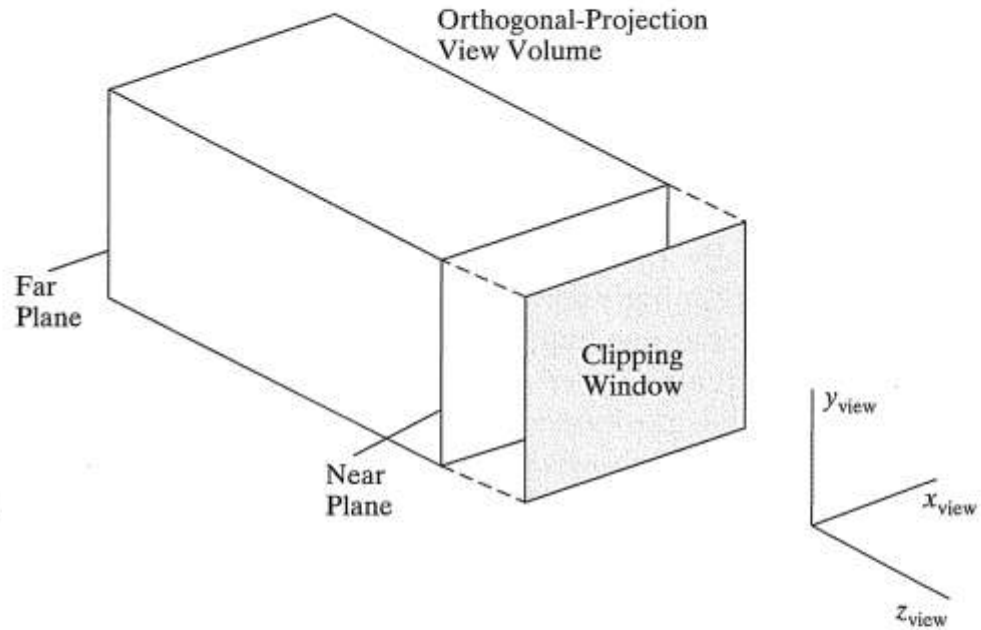


FIGURE 7-29 A finite orthogonal view volume with the view plane "in front" of the near plane.

Normalization Transformation for an Orthogonal Projection

Using an orthogonal transfer of coordinate positions onto the view plane, we obtain the projected position of any spatial point (x, y, z) as simply (x, y). Thus, once we have established the limits for the view volume, coordinate descriptions inside this rectangular parallelepiped are the projection coordinates, and they can be mapped into a **normalized view volume** without any further projection processing. Some graphics packages use a unit cube for this normalized view volume, with each of the x, y, and z coordinates normalized in the range from 0 to 1. Another normalization-transformation approach is to use a symmetric cube, with coordinates in the range from -1 to 1.

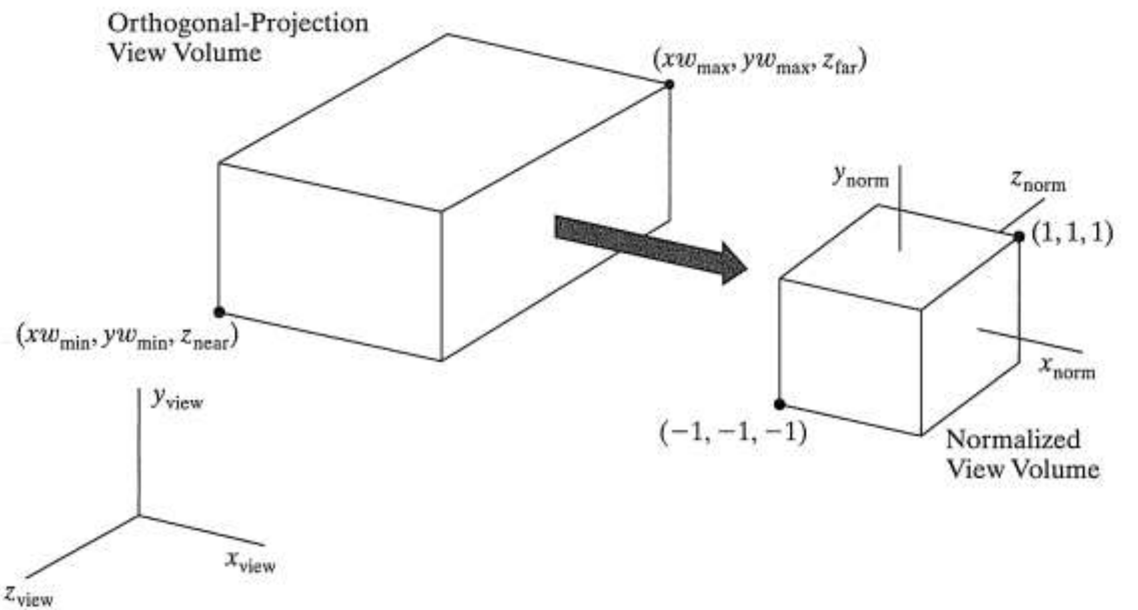


FIGURE 7-31
Normalization transformation from an orthogonal-projection view volume to the symmetric normalization cube within a left-handed reference frame.

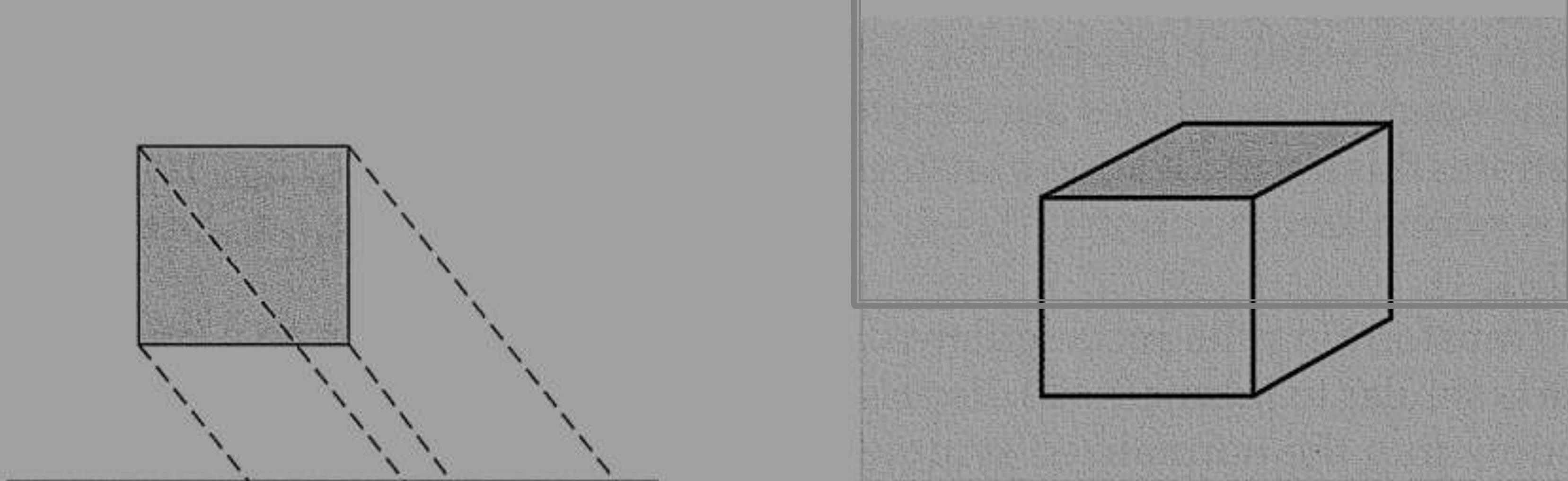
Therefore, the normalization transformation for the orthogonal view volume is

$$\mathbf{M}_{\text{ortho,norm}} = \begin{bmatrix} \frac{2}{xw_{\max} - xw_{\min}} & 0 & 0 & -\frac{xw_{\max} + xw_{\min}}{xw_{\max} - xw_{\min}} \\ 0 & \frac{2}{yw_{\max} - yw_{\min}} & 0 & -\frac{yw_{\max} + yw_{\min}}{yw_{\max} - yw_{\min}} \\ 0 & 0 & \frac{-2}{z_{\text{near}} - z_{\text{far}}} & \frac{z_{\text{near}} + z_{\text{far}}}{z_{\text{near}} - z_{\text{far}}} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7-7}$$

This matrix is multiplied on the right by the composite viewing transformation $\mathbf{R} \cdot \mathbf{T}$ (Section 7-4) to produce the complete transformation from world coordinates to normalized orthogonal-projection coordinates.

Oblique Parallel Projections in Drafting and Design

For applications in engineering and architectural design, an oblique parallel projection is often specified with two angles, α and ϕ, as shown in Fig. 7-33. A spatial position (x, y, z), in this illustration, is projected to (x_p, y_p, z_{vp}) on a view plane,



View Plane (a)

View Plane (b)

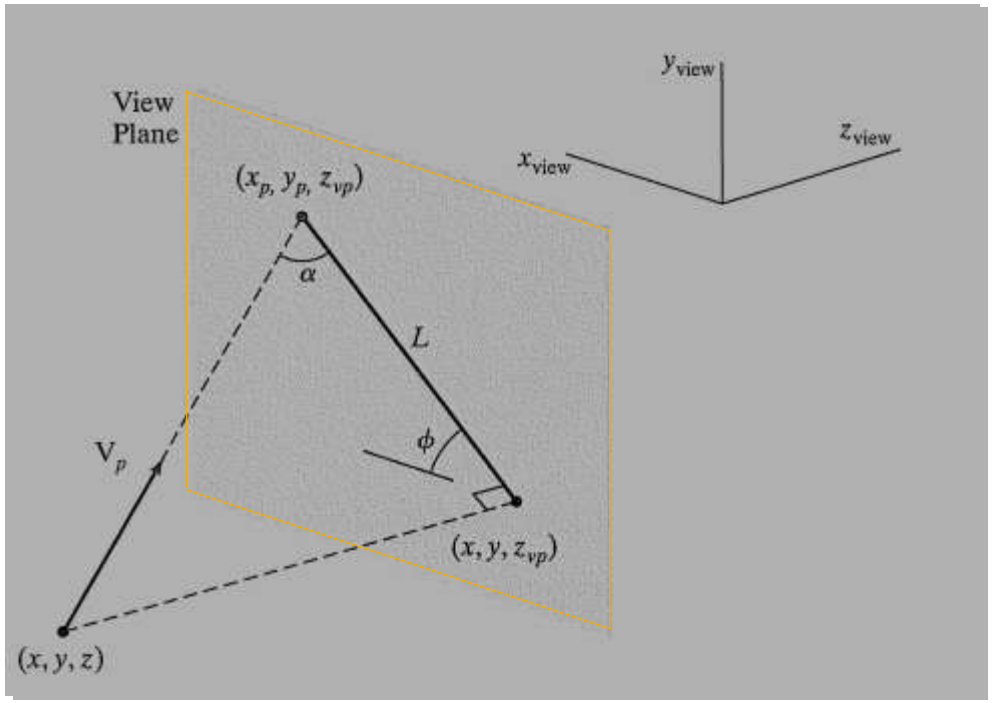


FIGURE 7-37 Oblique parallel projection of position (x, y, z) to a view plane along a projection line defined with vector $\mathbf{V}_p$.

$$x_p = x + L\cos\phi$$
$$y_p = y + L\sin\phi \tag{7-8}$$

Length L depends on the angle α and the perpendicular distance of the point (x, y, z) from the view plane:

$$\tan\alpha = \frac{z_{vp} - z}{L} \tag{7-9}$$

Thus

$$L = \frac{z_{vp} - z}{\tan\alpha}$$
$$= L_1(z_{vp} - z) \tag{7-10}$$

where $L_1 = \cot\alpha$, which is also the value of L when $z_{vp} - z = 1$. We can then write the oblique parallel projection equations 7-8 as

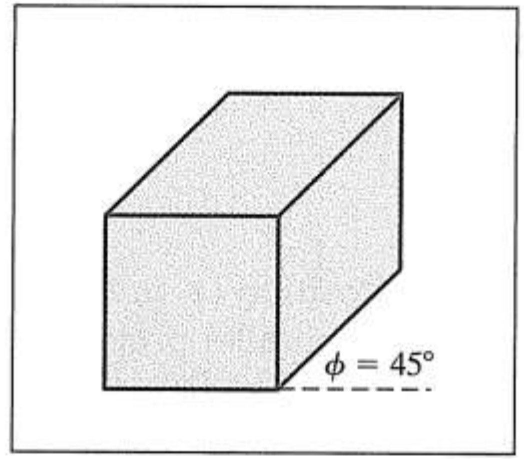
$$x_p = x + L_1(z_{vp} - z)\cos\phi$$
$$y_p = y + L_1(z_{vp} - z)\sin\phi \tag{7-11}$$

Cavalier and Cabinet Oblique Parallel Projections

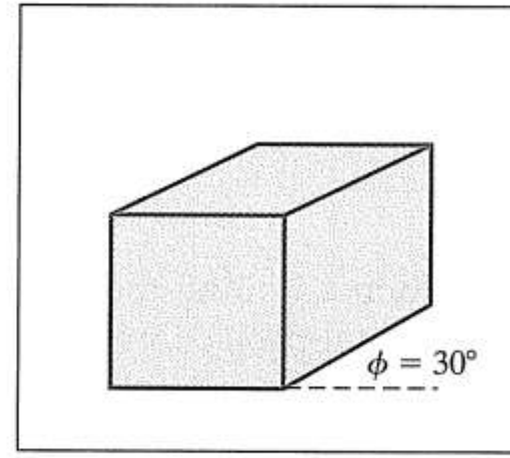
Typical choices for angle ϕ are 30° and 45°, which display a combination view of the front, side, and top (or front, side, and bottom) of an object. Two commonly used values for α are those for which $\tan\alpha = 1$ and $\tan\alpha = 2$. For the first case, $\alpha = 45°$ and the views obtained are called **cavalier** projections. All lines perpendicular to the projection plane are projected with no change in length. Examples of cavalier projections for a cube are given in Fig. 7-35.

When the projection angle α is chosen so that $\tan\alpha = 2$, the resulting view is called a **cabinet** projection. For this angle ($\approx 63.4°$), lines perpendicular to the viewing surface are projected at half their length. Cabinet projections appear more realistic than cavalier projections because of this reduction in the length of perpendiculars. Figure 7-36 shows examples of cabinet projections for a cube.

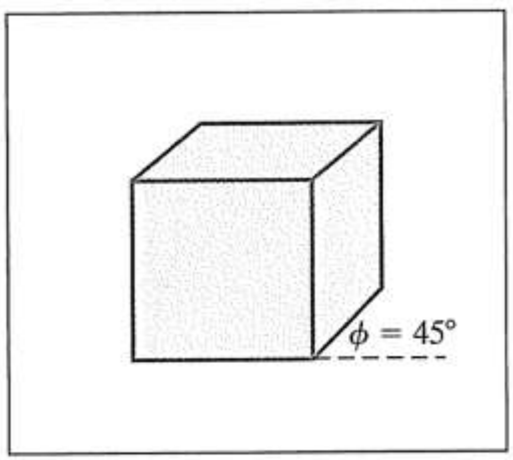
FIGURE 7-35 Cavalier projections of a cube onto a view plane for two values of angle ϕ. The depth of the cube is projected with a length equal to that of the width and height.



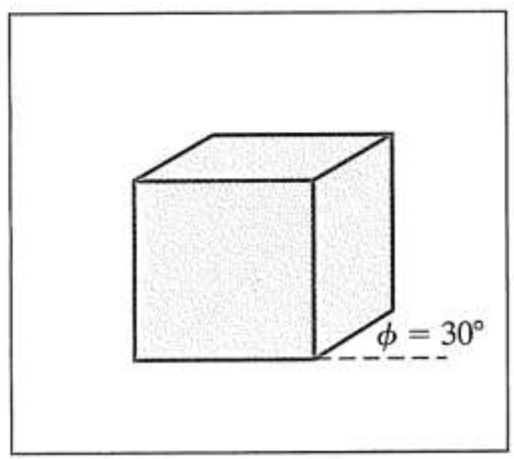
(a)



(b)



(a)



(b)

FIGURE 7-36 Cabinet projections of a cube onto a view plane for two values of angle ϕ. The depth is projected with a length that is one half that of the width and height of the cube.

END OF PRESENTATION