

Signals and Systems

For
Control & Communication
Departments

By
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References

- **Luis F. Chaparro,**” *Signals and systems using Matlab* ”, Elsevier,2011.
- **Oppenheim A. V., Willsky A.S. and Nawhab H.,**” *Signals and Systems* “, Prentice Hall, 2nd edition,1997.

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Chapter One

Introduction to Signals

Definition

Examples of signals

Classification of signals

Important signals (elementary signals)

Introduction of Signals

- The concept and theory of signals and systems are needed in almost all electrical , control, communications engineering fields and in many other engineering and scientific fields.
- **A signal** is a function representing a physical quantity or variable, and typically it contains information about the behaviour or nature of it.
- Mathematically, a signal is represented as a function of an independent variable t (time).
- Eg. a signal x is denoted by $x(t)$.

Some Examples of signals

- Electrical signals like voltages, current ,...
- Acoustic signals like audio or speech signals (analog or digital).
- Video signals like intensity variation in an image.
- Noise which will be treated as unwanted signal

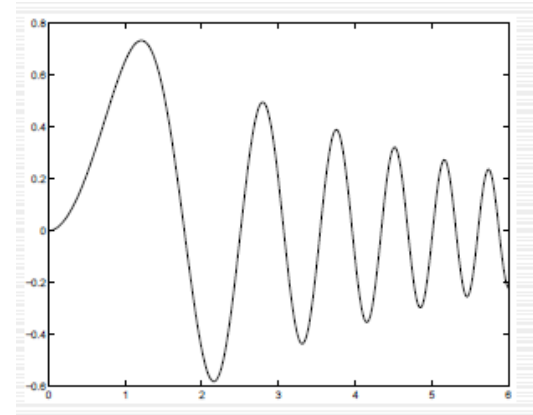
Signal classification

Signals can be classified as:

1. Continuous-time and Discrete-time
2. Energy and Power
3. Real and Complex
4. Periodic and Non-periodic
5. Analog and Digital
6. Even and Odd
7. Deterministic and Random

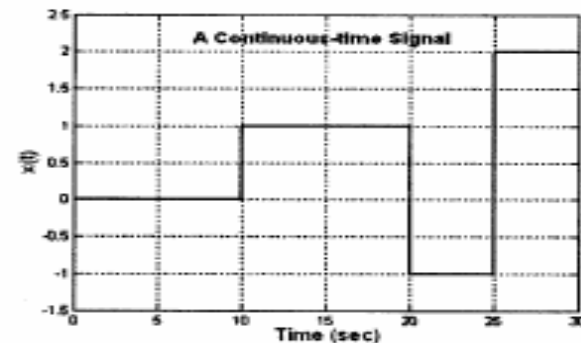
Continuous-Time and Discrete-Time Signals:

- A) Continuous Time Signal:
- A signal $x(t)$ is a continuous-time signal if t is a continuous variable



- ***A piecewise continuous-time signal***

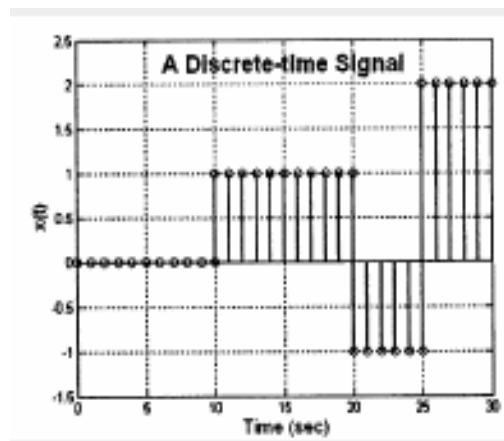
$$x(t) = \begin{cases} 0; & 0 \leq t \leq 10 \\ 1; & 10 < t \leq 20 \\ -1; & 20 \leq t \leq 25 \\ 2; & 25 < t \leq 30 \end{cases}$$



A discrete-time signal or DT signals

- *B) Discrete Time Signal*
- A discrete signal $x[n]$ is defined only at discrete instances. Thus, the independent variable has discrete values only. That is, the signal $x[n]$ is obtained by sampling $x(t)$ at times $t = nT_s$, where n is an integer and T_s is the *sampling period* or the time between samples.
- A piecewise discrete-time signal

$$x[n] = \begin{cases} 0; & 0 \leq n \leq 10 \\ 1; & 10 < n \leq 20 \\ -1; & 20 \leq n \leq 25 \\ 2; & 25 < n \leq 30 \end{cases}$$



$$x[n] = x(n\tau) \quad \forall n$$

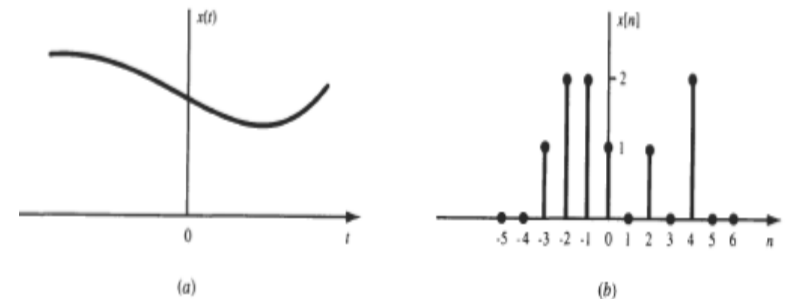


Fig. 1-1 Graphical representation of (a) continuous-time and (b) discrete-time signals.

2) Periodic and Non periodic of CT & DT signals

- A CT signal $x(t)$ is said to be *periodic* if it satisfies the following property:

$$x(t) = x(t + T_0),$$

- at all time t and for some positive constant T_0 . The smallest positive value of T_0 , is referred to as the *fundamental period* of $x(t)$.

Likewise,

- a DT signal $x[k]$ is said to be *periodic* if $x[k] = x[k + K_0]$

- at all time k and for some positive constant K_0
- The fundamental period of a signal is called the *fundamental frequency*. Mathematically, the fundamental frequency is expressed as follows:

$$f_0 = \frac{1}{T_0}, \text{ for CT signals, or } f_0 = \frac{1}{K_0}, \text{ for DT signals,}$$

- A signal that is not periodic is called an ***aperiodic or non-periodic signal***.

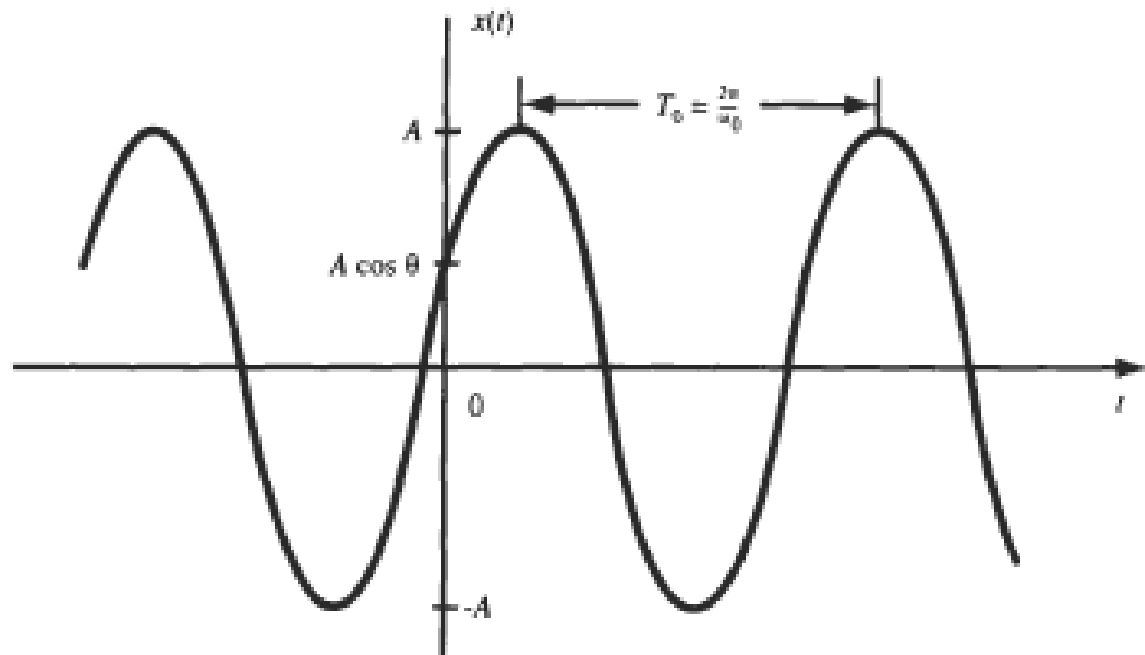
- For sinusoidal signal $x(t)$: $x(t) = A \sin(\omega_0 t + \theta)$.

- where A is the amplitude (real), ω_0 is the radian frequency in radians per second, and θ is the phase angle in radians. The sinusoidal signal $x(t)$ is shown in figure below, is periodic with fundamental period T_0 , and the fundamental frequency f_0 :

$$T_0 = \frac{2\pi}{\omega_0}$$

$$f_0 = \frac{1}{T_0} \text{ hertz (Hz)}$$

$$\omega_0 = 2\pi f_0$$



Examples

- (i) CT sine wave: $x_1(t) = \sin(4\pi t)$ is a periodic signal with period $T_1 = 2\pi/4\pi = 1/2$;
- (ii) CT cosine wave: $x_2(t) = \cos(3\pi t)$ is a periodic signal with period $T_2 = 2\pi/3\pi = 2/3$;
- (vi) CT linear relationship: $x_3(t) = 2t + 5$ is an aperiodic signal.

Real and Complex

- A signal $x(t)$ is a real signal if its value is a real number, and a signal $x(t)$ is a complex signal if its value is a complex number.

A general complex signal $x(t)$ is a function of the form

$$x(t) = x_1(t) + jx_2(t)$$

$$\text{Where } j = \sqrt{-1} \quad \text{Re}\{x(t)\} = x_1(t) \quad \text{Im}\{x(t)\} = x_2(t)$$

- Magnitude or absolute value is given by:

$$|x(t)| = \sqrt{x_1^2(t) + x_2^2(t)} = \sqrt{x_1(t)x_1^*(t)}$$

- Phase or angle $\angle x(t) = \tan^{-1} \left\{ \frac{x_2(t)}{x_1(t)} \right\}$

Analog and Digital

- If a continuous-time signal can take on any value in the continuous interval (a, b) , where a, b , may be $-\infty$ and $+\infty$, then the continuous-time signal $x(t)$ is called an analog signal.
- If a discrete-time signal $x[n]$ can take on only a finite number of distinct values, then we call this signal a digital signal.
- **Digital signal** is discrete-time signal whose values belong to a defined set of real numbers

$$x[n] = x(t_n) = a_i; \quad 1 \leq i \leq N$$

- **Binary signal** is digital signal whose values are 1 or 0

$$x[n] = x(t_n) = 0 \text{ or } 1; \quad \forall n$$

- **Analog signal** is a non-digital signal

Even and Odd

- Even Signals

- The CT signal $x(t)$ & DT signal $x[n]$ is an even signal if it satisfies the condition

$$x(t) = x(-t); \quad \forall t \quad x[n] = x[-n]; \quad \forall n$$

- Even signals are **symmetric about the vertical axis**

- Odd Signals

- The signal is said to be an odd signal if it satisfies the condition

$$x(t) = -x(-t); \quad \forall t \quad x[n] = -x[-n]; \quad \forall n$$

- Odd signals are **symmetric about the time origin.**

Facts on Even and Odd signals

- Product of 2 even or 2 odd signals is an **even signal**
- Product of an even and an odd signal is an **odd signal**
- Any signal (continuous and discrete) can be expressed as **sum of an even and an odd signal, or Even and odd decomposition**

$$\begin{aligned}x(t) &= x_e(t) + x_o(t); & x[n] &= x_e[n] + x_o[n] \\x_e(t) &= \frac{x(t) + x(-t)}{2}; & x_o(t) &= \frac{x(t) - x(-t)}{2} \\x_e[n] &= \frac{x[n] + x[-n]}{2}; & x_o[n] &= \frac{x[n] - x[-n]}{2}\end{aligned}$$

Energy and Power Signals in CT & DT signals

The *instantaneous power* (Kjoule/sec=watt, *قدرة*) at time $t = t_0$ of a **real**-valued CT signal $x(t)$ is given by $x^2(t_0)$. Similarly, the instantaneous power of a real-valued DT signal $x[k]$ at time instant $k = k_0$ is given by $x^2[k]$.

If the signal is **complex-valued**, the expressions for the instantaneous power are modified to $|x(t_0)|^2$ or $|x[k_0]|^2$, where the **symbol $|\cdot|$** represents the absolute value of a complex number.

A signal $x(t)$, or $x[k]$, is called **an energy signal** (*K Joule, طاقة*) if the total energy E_x has a non-zero finite value, i.e. $0 < E_x < \infty$.

On the other hand, a signal is called a **power signal** if it has non-zero finite power, i.e. $0 < P_x < \infty$.

Note that a signal cannot be both an energy and a power signal simultaneously. The energy signals have zero average power whereas the power signals have infinite total energy. Some signals, however, can be classified as neither power signals nor as energy signals.

**Periodic signals are power signals;
nonperiodic signals are energy signals.**

When both power and energy are infinite, the signal is neither a power nor an energy signal. As a matter of fact, a true power signal cannot exist in the real world because it would require a power source that operates for an infinite amount of time.

The square root of the average power of a power signal is what is usually defined as the RMS value of that signal.

A) Energy

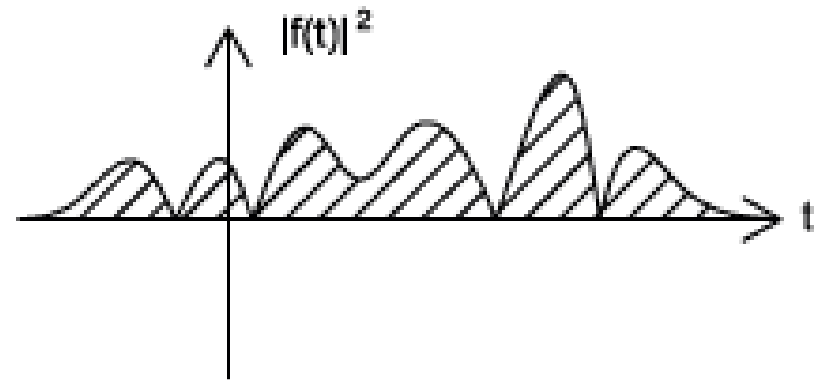
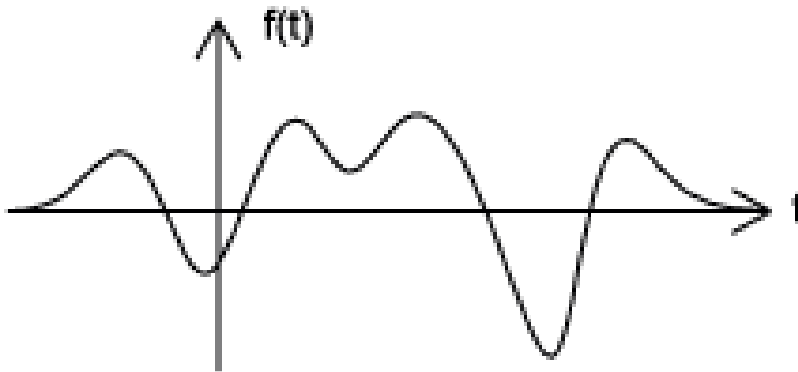
The *energy* present in a CT or DT signal within a given time interval is given by the following:

CT signals

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt;$$

DT sequences

$$E_x = \sum_{k=-\infty}^{\infty} |x[k]|^2.$$



CT signals $E_{(T_1, T_2)} = \int_{T_1}^{T_2} |x(t)|^2 dt$ in interval $t = (T_1, T_2)$ with $T_2 > T_1$;

DT sequences $E_{[N_1, N_2]} = \sum_{k=N_1}^{N_2} |x[k]|^2$ in interval $k = [N_1, N_2]$ with $N_2 > N_1$.

B)Power

Since **power is defined as energy per unit time**, the *average power* of a CT signal $x(t)$ over the interval $t = (-\infty, \infty)$ and of a DT signal $x[k]$ over the range $k = [-\infty, \infty]$ are expressed as follows:

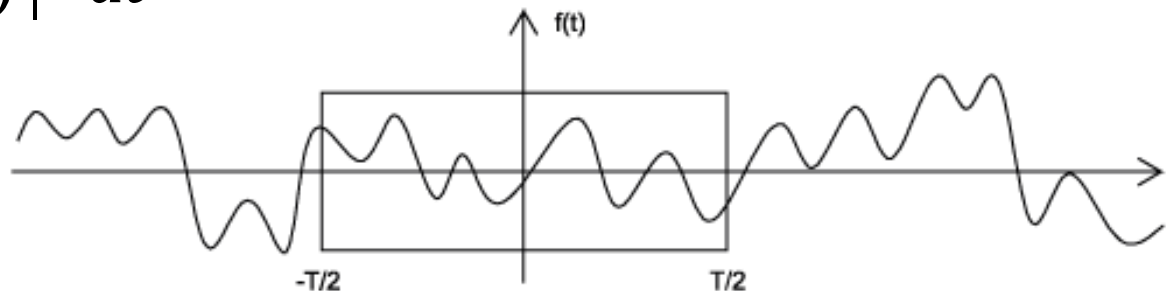
CT signals

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt.$$

DT sequences

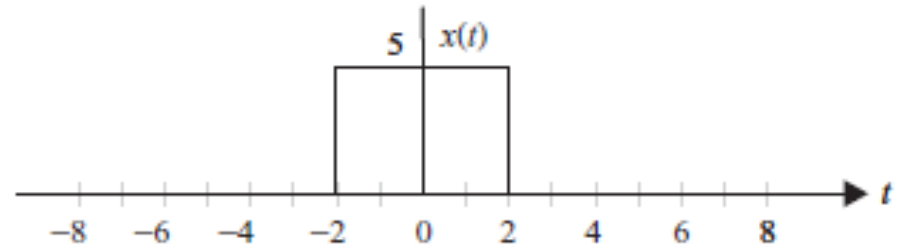
$$P_x = \frac{1}{2K + 1} \sum_{k=-K}^K |x[k]|^2.$$

$$P_{\text{Periodic } f} = \frac{1}{T} \int_{t_0}^{T+t_0} |f(t)|^2 dt$$



Example : Consider the CT signals shown .calculate the average power, and energy. Classify the signals as power or energy signals.

$$x(t) = \begin{cases} 5 & -2 \leq t \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$



energy

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-2}^2 25 dt = 100;$$

average power

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{T} E_x = 0.$$

Because $x(t)$ has finite energy ($0 < E_x = 100 < \infty$) it is an energy signal.

Homework: determine if the following signal s are Energy signal, Power signal, or neither, and evaluate E and P for each signal

$$a(t) = 3\sin(2\pi t), \quad -\infty < t < \infty$$

This is a periodic signal, so it must be a power signal. Let us prove it.

$$\begin{aligned} E_a &= \int_{-\infty}^{\infty} |a(t)|^2 dt = \int_{-\infty}^{\infty} |3\sin(2\pi t)|^2 dt \\ &= 9 \int_{-\infty}^{\infty} \frac{1}{2} [1 - \cos(4\pi t)] dt \\ &= 9 \int_{-\infty}^{\infty} \frac{1}{2} dt - 9 \int_{-\infty}^{\infty} \cos(4\pi t) dt \\ &= \infty \quad \text{J} \end{aligned}$$

Since $a(t)$ is periodic with period $T = 2\pi/2\pi = 1$ second, we get

$$\begin{aligned}
 P_a &= \frac{1}{1} \int_0^1 |a(t)|^2 dt = \int_0^1 |3\sin(2\pi t)|^2 dt \\
 &= 9 \int_0^1 \frac{1}{2} [1 - \cos(4\pi t)] dt \\
 &= 9 \int_0^1 \frac{1}{2} dt - 9 \int_0^1 \cos(4\pi t) dt \\
 &= \frac{9}{2} - \left[\frac{9}{4\pi} \sin(4\pi t) \right]_0^1 \\
 &= \frac{9}{2} \text{ W}
 \end{aligned}$$

$$b(t) = 5e^{-2|t|}, \quad -\infty < t < \infty$$

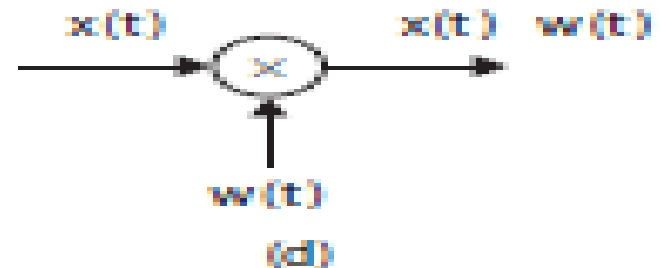
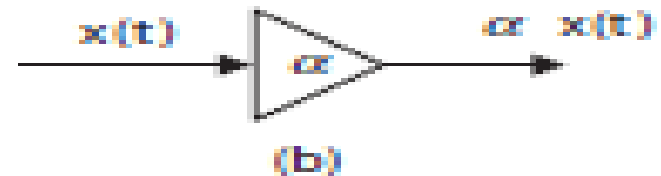
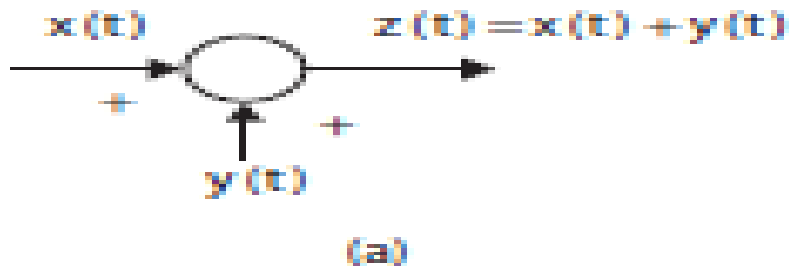
$$c(t) = \begin{cases} 4e^{+3t}, & |t| \leq 5 \\ 0, & |t| > 5 \end{cases}$$

Deterministic and Random signal

- A signal is **deterministic** whose future values can be predicted accurately. Deterministic signals are those signals whose values are completely specified for any given time. Thus, a deterministic signal can be modelled by a known function of time t .
- **Random** signals are those signals that take random values at any given time and must be characterized statistically. A signal is random whose future values can NOT be predicted with complete accuracy.

Basic Signal Operations:

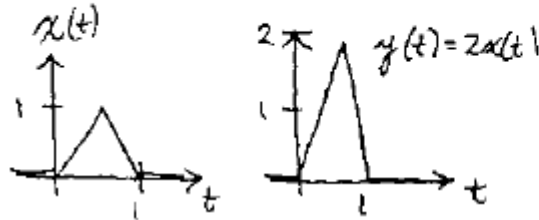
- *Signal addition - Constant multiplication-
Time and frequency shifting*



Operations on Signals

1) Amplitude scaling

AMPLITUDE SCALING



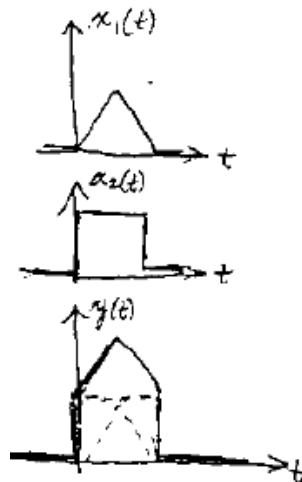
$$y(t) = C x(t)$$

SCALING
FACTOR

E.G., AMPLIFIER, RESISTOR

2) Addition between signals

ADDITION



$$y(t) = x_1(t) + x_2(t)$$

E.G., AUDIO MIXER

"ADDITION IS POINTWISE, AT EACH t "

MULTIPLICATION

$$y(t) = x_1(t) x_2(t)$$

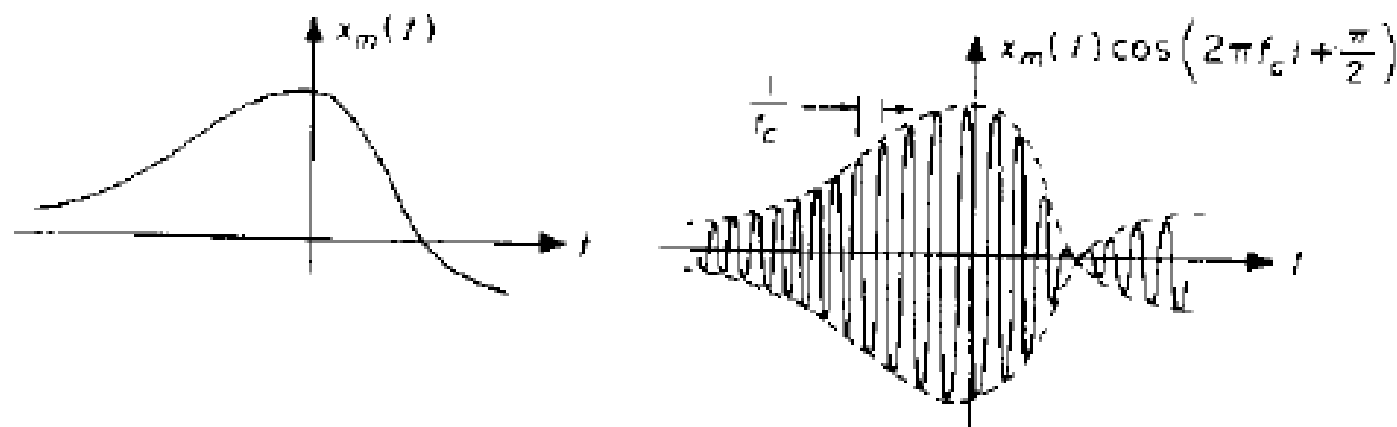


Figure 17.1-2. An amplitude-modulated carrier.

DIFFERENTIATION

$$y(t) = \frac{d}{dt} x(t)$$

$$\text{E.G., } v(t) = L \frac{d}{dt} i(t)$$

INTEGRATION

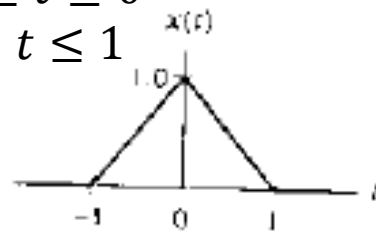
$$y(t) = \int_{-\infty}^t x(\tau) d\tau$$

$$\text{E.G., } v(t) = \frac{1}{C} \int_{-\infty}^t i(\tau) d\tau$$

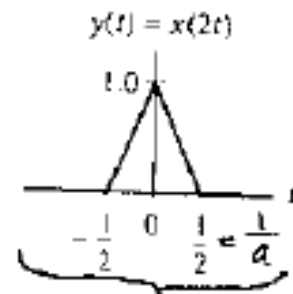
TIME SCALING

$$y(t) = x(at)$$

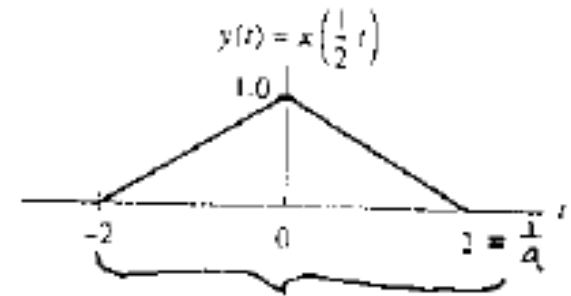
$$x(t) = \begin{cases} t+1 & -1 \leq t \leq 0 \\ 1-t & 0 \leq t \leq 1 \end{cases}$$



(a)



$a > 1$, COMPRESSION

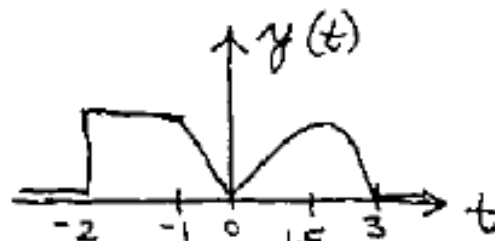
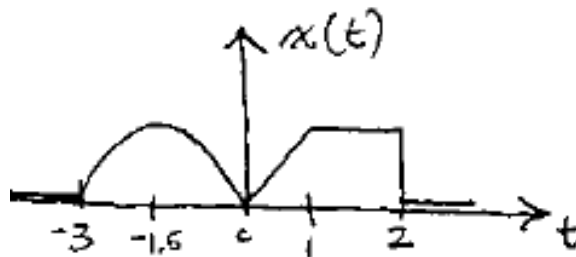


$0 < a < 1$, EXPANSION

FIGURE 1.19 Time-scaling operation: (a) continuous-time signal $x(t)$, (b) compressed version of $x(t)$ by a factor of 2, and (c) expanded version of $x(t)$ by a factor of 2.

REVERSAL TIME REFLECTION

$$y(t) = x(-t)$$

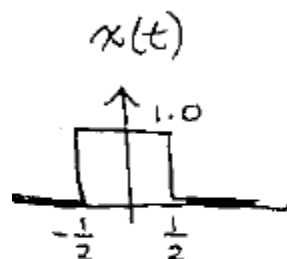


USES:
CONVOLUTION,
FILTERING

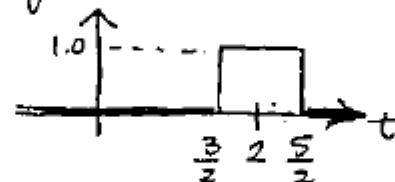
TIME SHIFTING

$$y(t) = x(t - t_0)$$

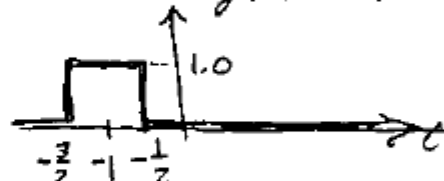
⚡
TIME SHIFT



$$y(t) = x(t - 2)$$



$$z(t) = x(t + 1)$$



$t_0 > 0$: SHIFTED RIGHT, TIME DELAYED

$t_0 < 0$: SHIFTED LEFT, TIME ADVANCED

Note that:

$$x(t) = \begin{cases} 1 & -1/2 \leq t \leq 1/2 \\ 0 & \text{otherwise} \end{cases}$$

ALL THE FOREGOING APPLIES TO DISCRETE-TIME...

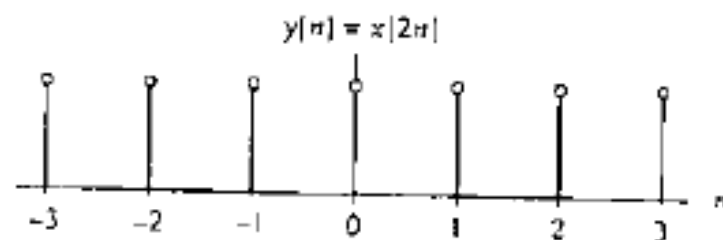
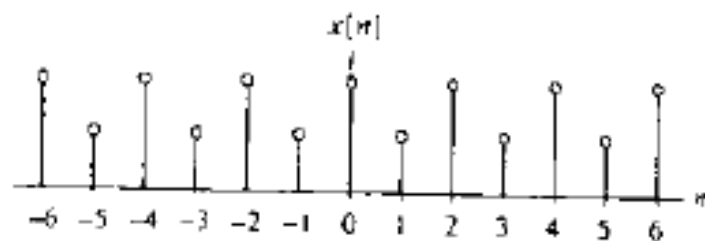
AMPLITUDE SCALING: $y[n] = c x[n]$
ADDITION: $y[n] = x_1[n] + x_2[n]$
MULTIPLICATION: $y[n] = x_1[n] x_2[n]$

} PTWISE FOR EACH n

REFLECTION: $y[n] = x[-n]$

TIME-SHIFTING: $y[n] = x[n-m]$

TIME-SCALING: $y[n] = x[kn]$



Elementary signals

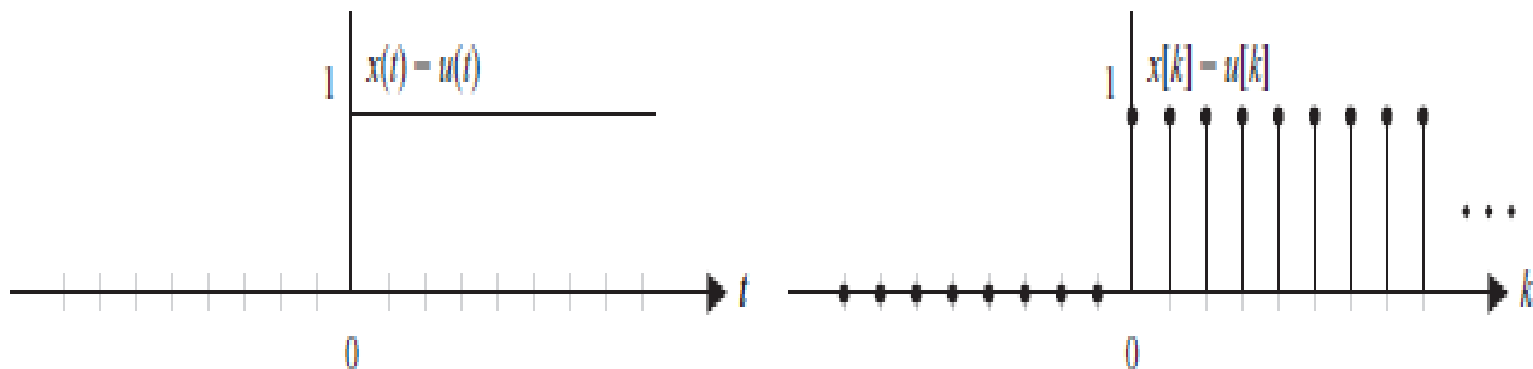
Elementary signals

1-Unit step Function:

- The CT unit step function $u(t)$ is defined as follows:
- The DT unit step function $u[k]$ is defined as follows:

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0. \end{cases}$$

$$u[k] = \begin{cases} 1 & k \geq 0 \\ 0 & k < 0. \end{cases}$$



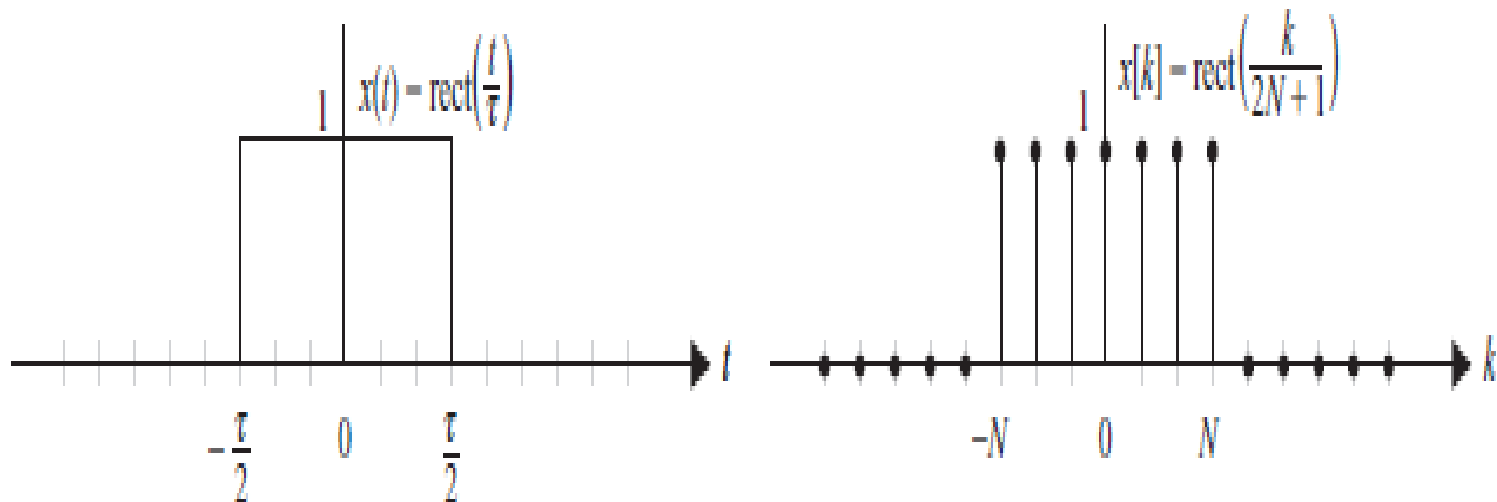
2) Rectangular pulse function:

- The CT rectangular pulse, $\text{rect}(t/\tau)$ is defined as follows

$$\text{rect}\left(\frac{t}{\tau}\right) = \begin{cases} 1 & |t| \leq \tau/2 \\ 0 & |t| > \tau/2 \end{cases}$$

- The DT rectangular pulse $\text{rect}(k/(2N + 1))$ is

$$\text{rect}\left(\frac{k}{2N + 1}\right) = \begin{cases} 1 & |k| \leq N \\ 0 & |k| > N \end{cases}$$

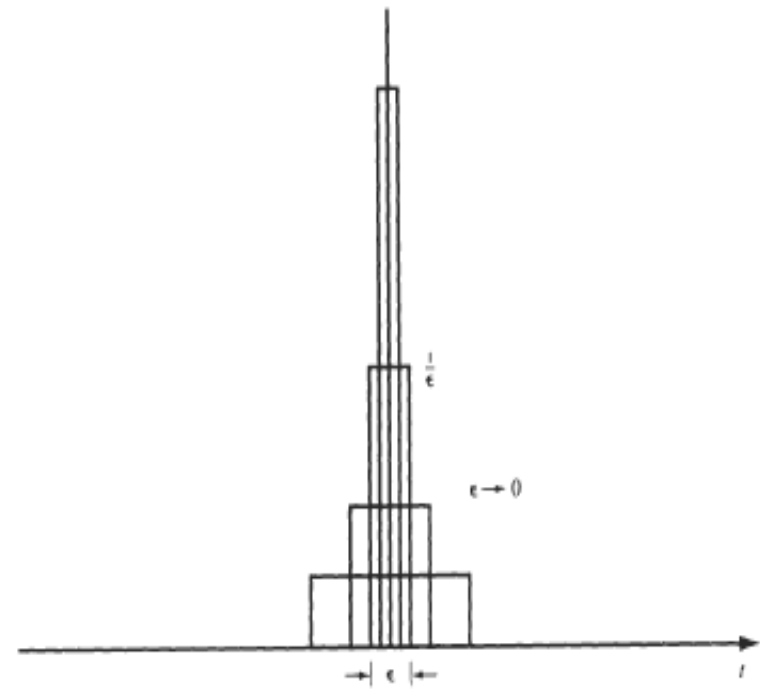
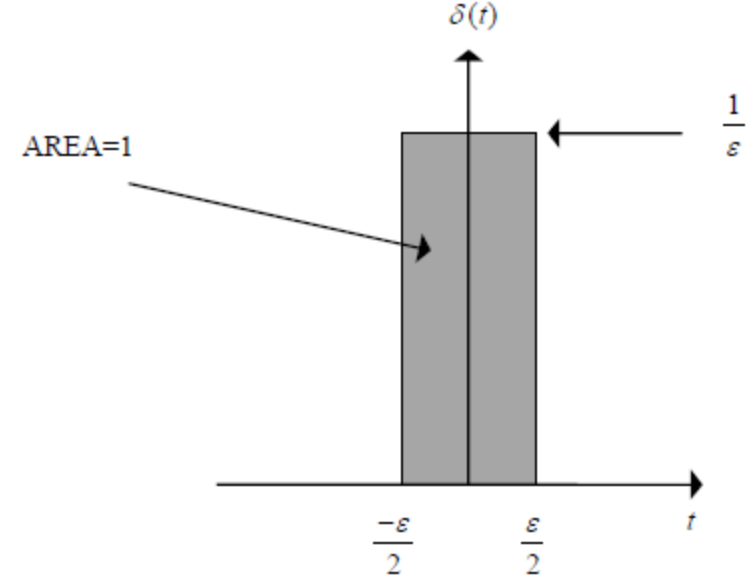
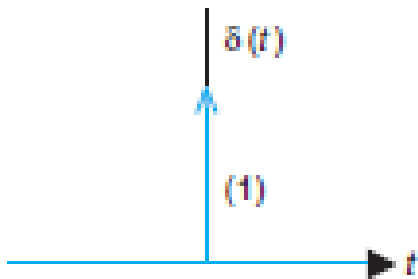


Or as shown in the figure , $\lim_{\epsilon \rightarrow 0}$ then $1/\epsilon \rightarrow \infty$, the signal is called **unit impulse** signal $\delta(t)$ as shown below:

$$\int_{-\infty}^{+\infty} \delta(t - \tau) dt = \int_{-\infty}^{+\infty} \delta(t) dt = 1$$

$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases}$$

$$\int_{-\epsilon}^{\epsilon} \delta(t) dt = 1$$



The unit step function can be related to the unit impulse function as follows.

$$u(t) = \int_{-\infty}^t \delta(\tau) d\tau \qquad \delta(t) = u'(t) = \frac{du(t)}{dt}$$

The same way in DT signal

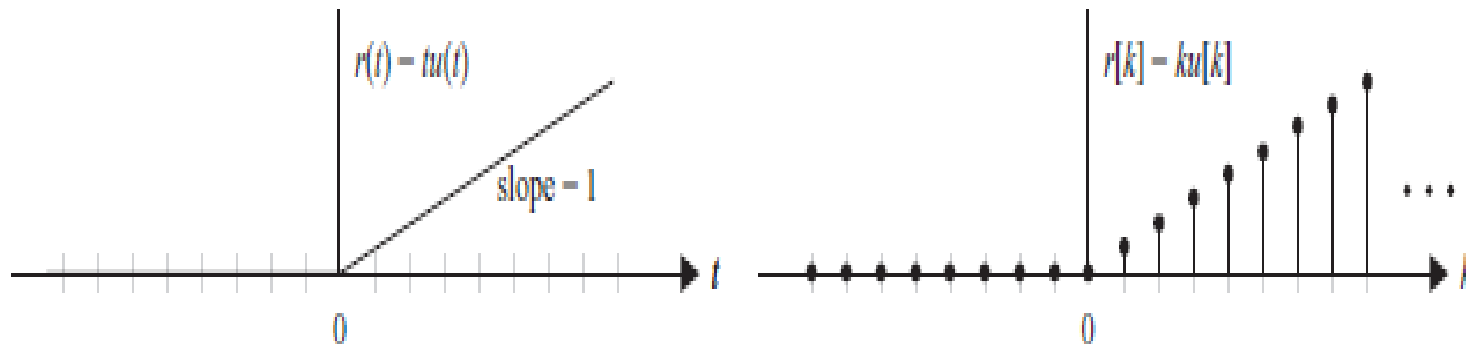
$$u[n] = \sum_{k=-\infty}^n \delta[k] = \sum_{k=0}^{+\infty} \delta[n-k]$$
$$\delta[n] = u[n] - u[n-1]$$

3) The Ramp function $r(t)$:

- The CT ramp function $r(t)$ is defined as follows:
- DT ramp function $r[k]$ is defined as follows:

$$r(t) = t u(t)$$

$$r[k] = ku[k] = \begin{cases} k & k \geq 0 \\ 0 & k < 0, \end{cases}$$



Its relation to the unit-step and the unit-impulse signals is

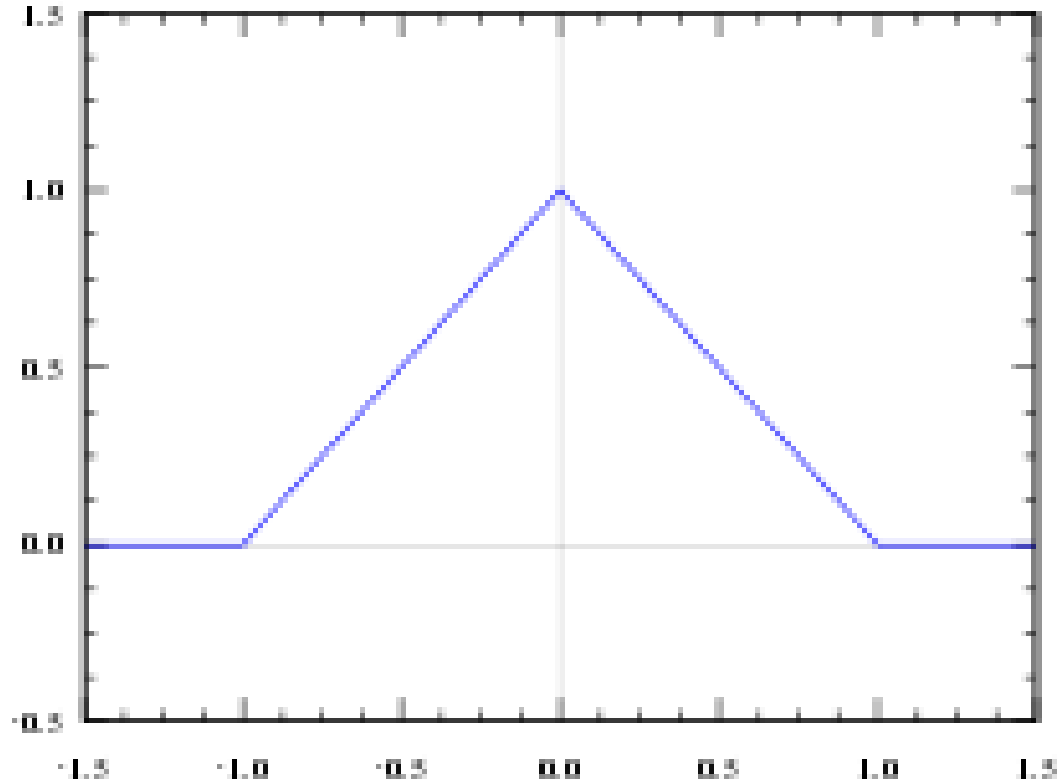
$$\frac{dr(t)}{dt} = u(t)$$

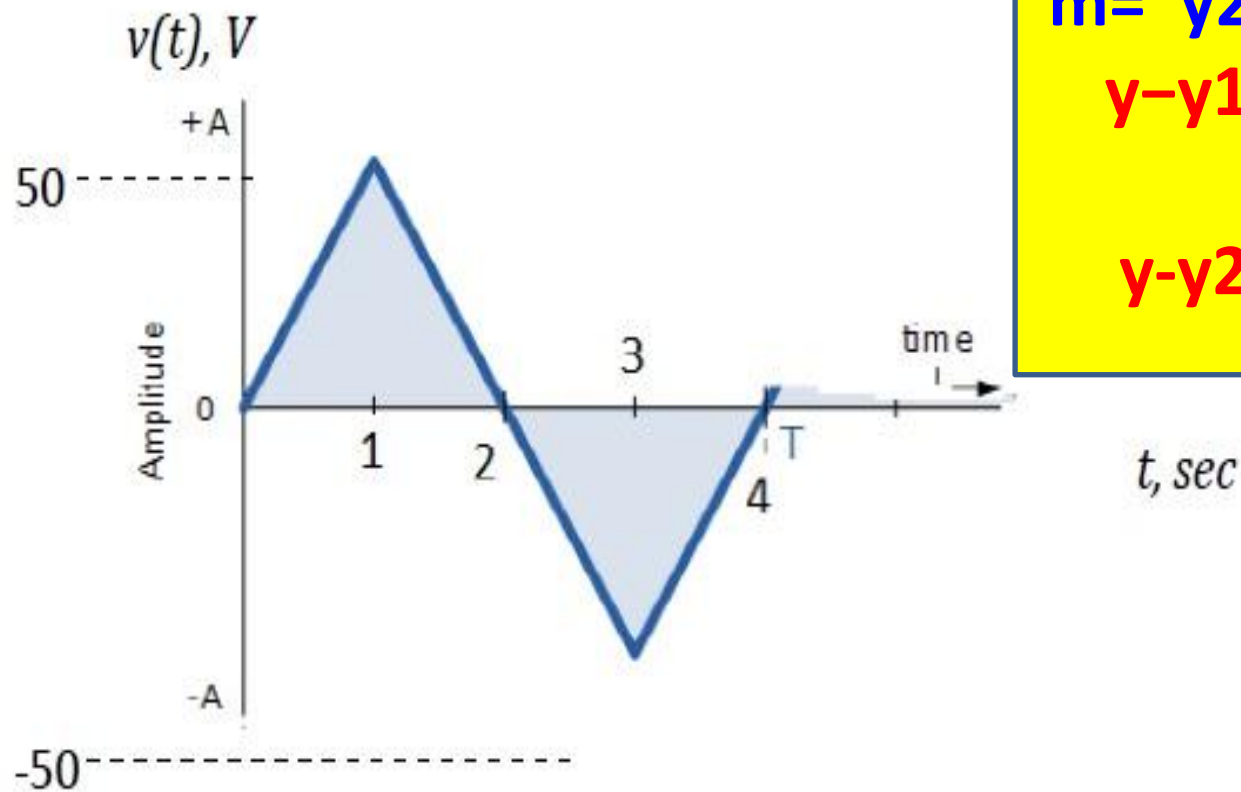
$$\frac{d^2r(t)}{dt^2} = \delta(t)$$

4) The Triangular signal tri :

- The triangular signal can be define as:

$$\text{tri}\left(\frac{t}{a}\right) = \begin{cases} 1 - t / a & |t| < |a| \\ 0 & \text{otherwise} \end{cases}$$





$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y - y_1 = m(x - x_1)$$

Or

$$y - y_2 = m(x - x_2)$$

Figure Q1

[Answer:
$$v(t) = \begin{cases} 50t & , 0 < t < 1 \\ -50t + 100 & , 1 < t < 3 \\ 50t - 200 & , 3 < t < 4 \end{cases} \text{ , triangular wave, } T = 4\text{s}]$$

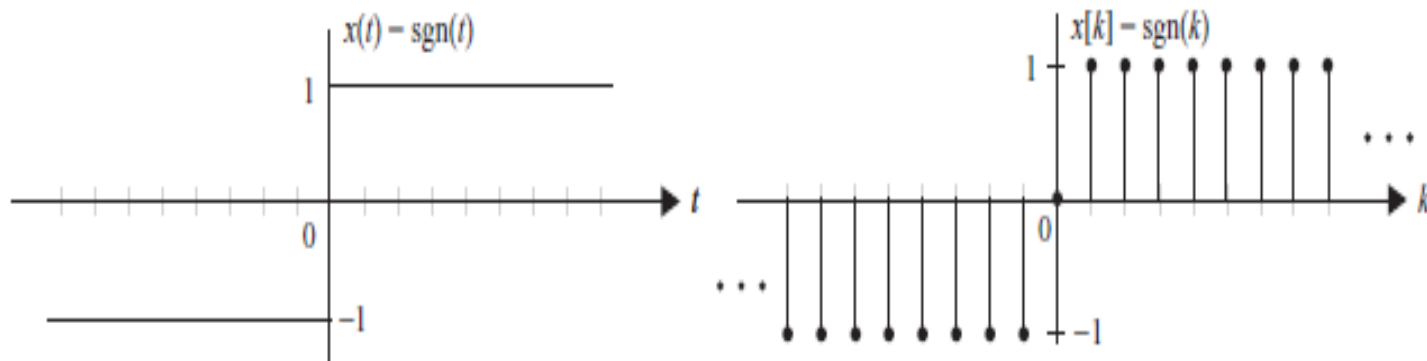
5) Signum function:

The *signum* (or *sign*) function, denoted by $\text{sgn}(t)$, is defined as follows:

$$\text{sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0. \end{cases}$$

- DT signum function is:

$$\text{sgn}[k] = \begin{cases} 1 & k > 0 \\ 0 & k = 0 \\ -1 & k < 0 \end{cases}$$



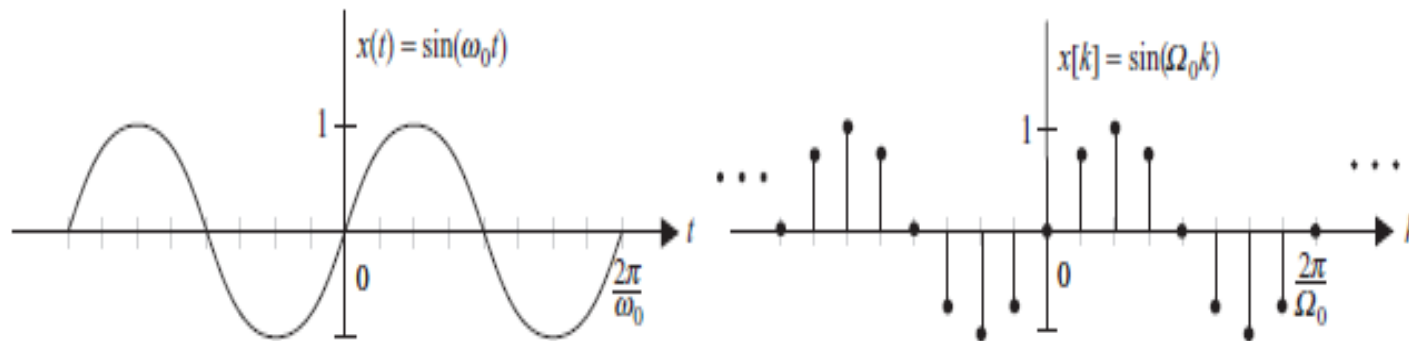
6) Sinusoidal function:

The CT sinusoid of frequency f_0 (or, equivalently, an angular frequency $\omega_0 = 2\pi f_0$) is defined as follows:

$$x(t) = \sin(\omega_0 t + \theta) = \sin(2\pi f_0 t + \theta),$$

The DT sinusoid is defined as follows:

$$x[k] = \sin(\Omega_0 k + \theta) = \sin(2\pi f_0 k + \theta),$$



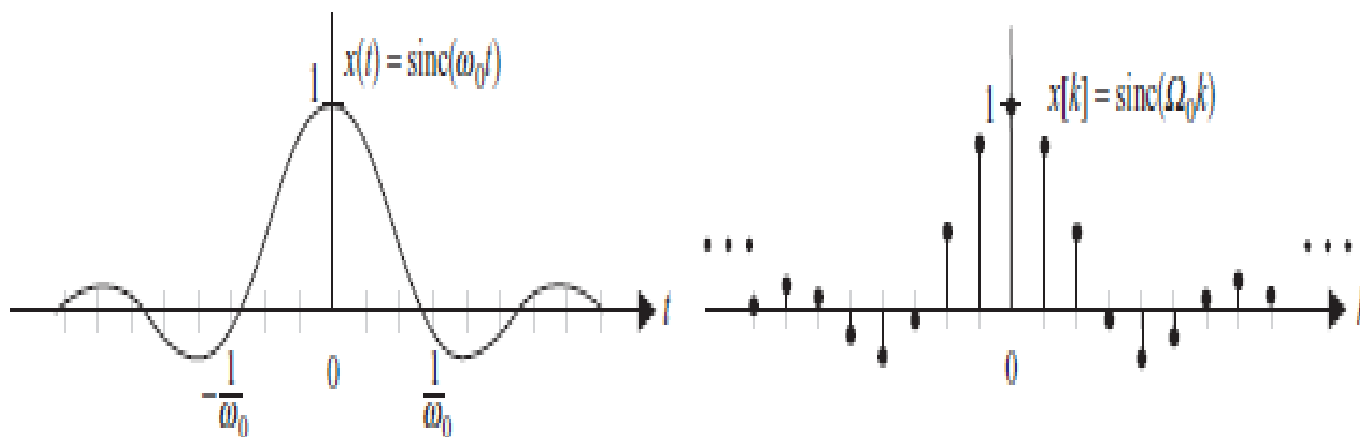
7) Sinc function:

The CT sinc function is defined as follows:

$$\text{sinc}(\omega_0 t) = \frac{\sin(\pi \omega_0 t)}{\pi \omega_0 t},$$

The DT sinc function is defined as follows:

$$\text{sinc}(\Omega_0 k) = \frac{\sin(\pi \Omega_0 k)}{\pi \Omega_0 k},$$



Examples

- 1) Consider a discrete signal $x[n]$ defined as shown, find $y[n]$ if $y[n]=x[3n-2]$

$$x[n] = \begin{cases} 1 & -2 \leq n \leq 2 \\ 0 & |n| > 2 \end{cases}$$

n	-2	-1	0	1	2
3n-2	-8	-5	-2	1	4
X[3n-2]	0	0	1	1	0
Y[n]	0	0	1	1	0

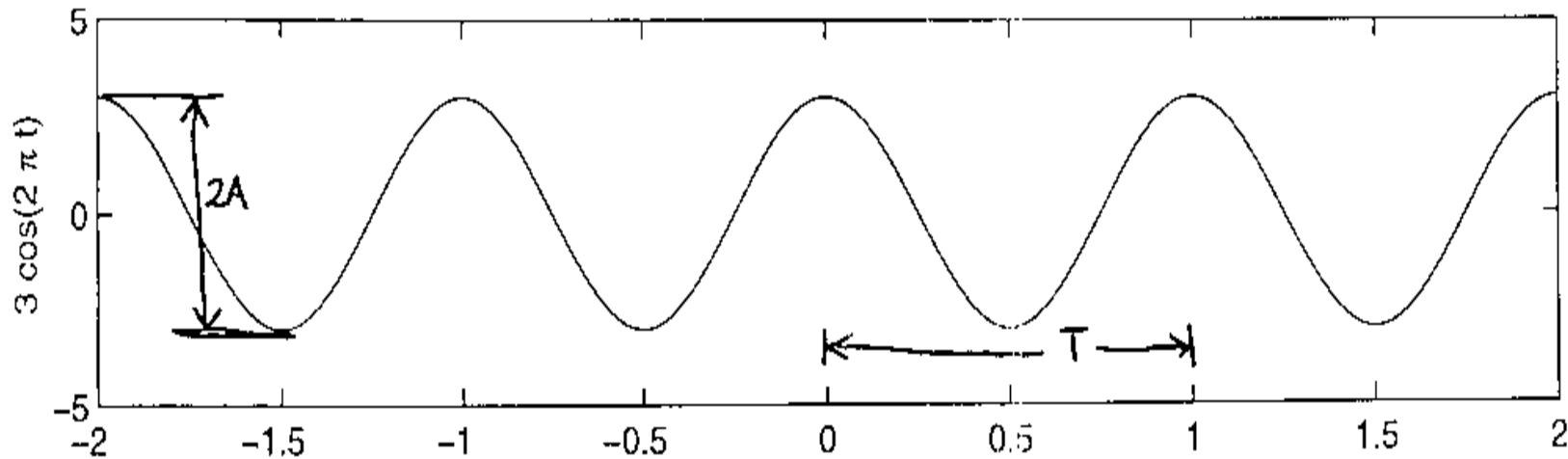
$$y[n] = \begin{cases} 1 & \text{for } n = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

I- example

2) Consider the sinusoidal signals, find T for each of them:

$$X(t) = 3 \cos(2\pi t),$$

$x(t) = 3 \cos(4\pi t)$ as homework



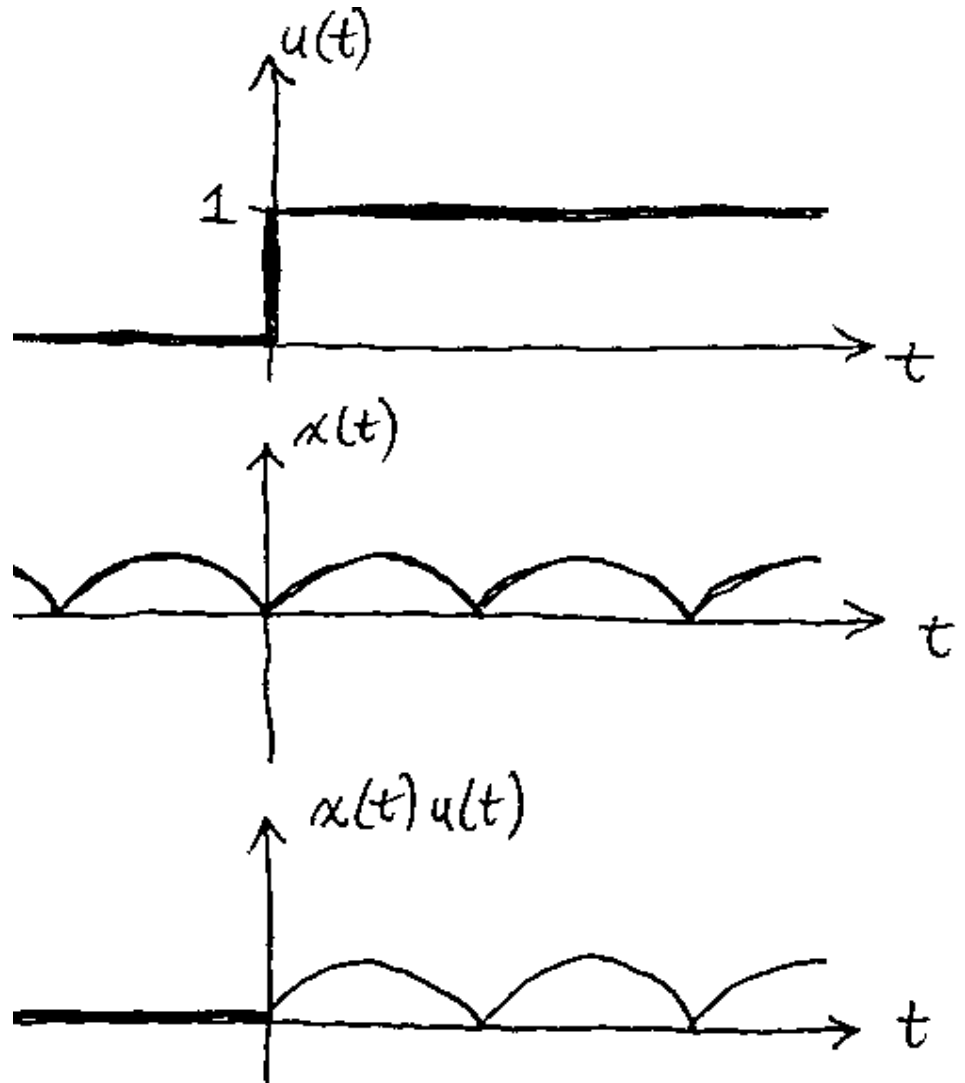
$$\begin{aligned} T &= \frac{2\pi}{\omega} \\ &= \frac{2\pi}{2\pi} \\ &= 1 \end{aligned}$$

3) It is given a unit step function $u(t)$, and $x(t)$ as shown, what will be the shape of $x(t)u(t)$

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$k u(t) = \begin{cases} k & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$x(t) u(t) = \begin{cases} x(t) & t \geq 0 \\ 0 & t < 0 \end{cases}$$



4) Given the function $x(t)$:

$$x(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- Find mathematical expressions for $x(t)$ delayed by 2, advanced by 2, and the reflected signal $x(-t)$.

Solution

- The delayed signal $x(t-2)$, the value $x(0)$ (which in $x(t)$ occurs at $t = 0$) in $x(t-2)$ now occurs when $t = 2$,

$$x(t-2) = \begin{cases} 1 & 0 \leq t-2 \leq 1 \text{ or } 2 \leq t \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t+2) = \begin{cases} 1 & 0 \leq t+2 \leq 1 \text{ or } -2 \leq t \leq -1 \\ 0 & \text{otherwise} \end{cases}$$

$$x(-t) = \begin{cases} 1 & 0 \leq -t \leq 1 \text{ or } -1 \leq t \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

I- example

Example 5 : Express the CT signal $x(t)$ as a combination of an even signal and an odd signal.

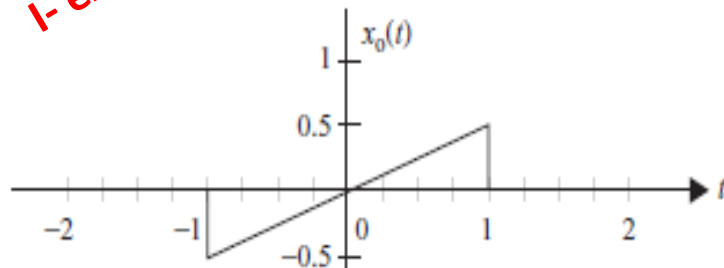
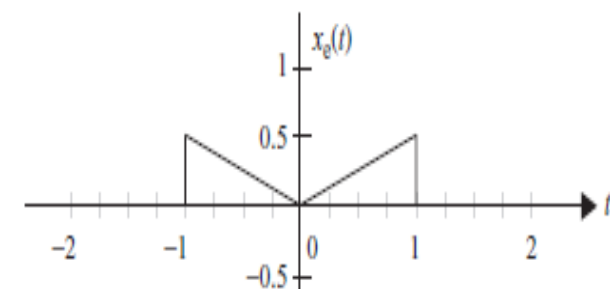
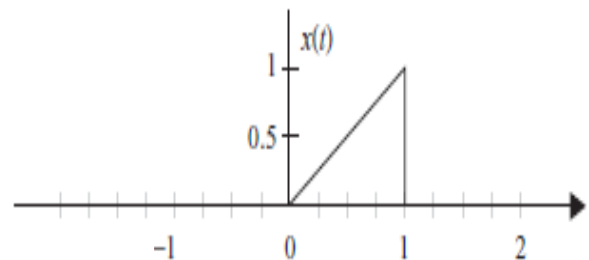
$$x(t) = \begin{cases} t & 0 \leq t < 1 \\ 0 & \text{elsewhere} \end{cases}$$

- First find $x(-t)$, which is given by:

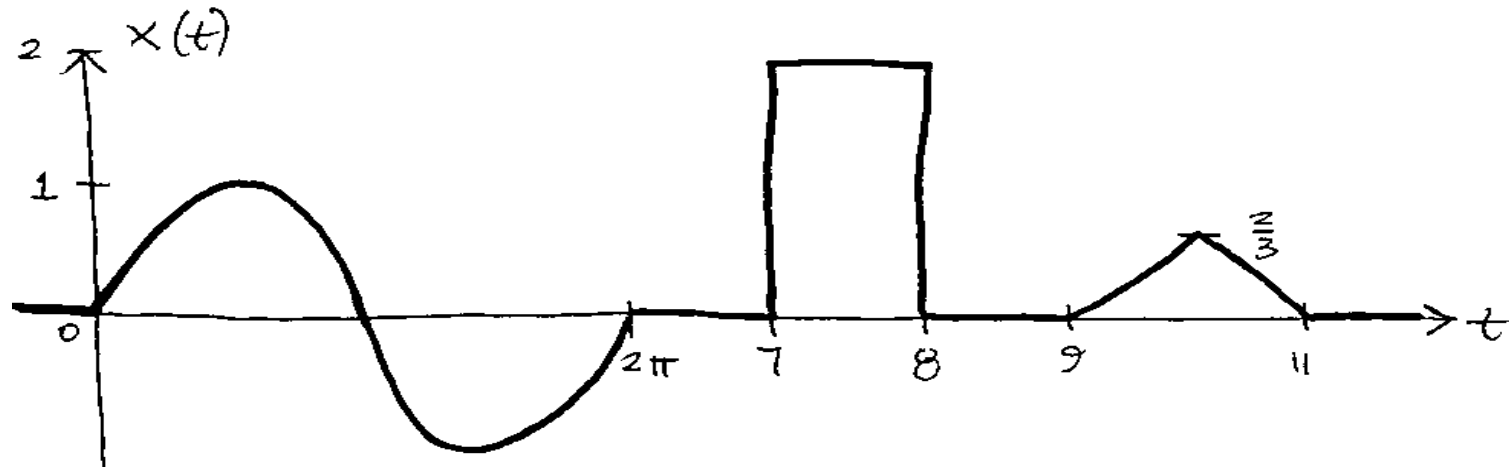
$$x(-t) = \begin{cases} -t & 0 \leq -t < 1 \\ 0 & \text{elsewhere} \end{cases} = \begin{cases} -t & -1 < t \leq 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)] = \begin{cases} \frac{1}{2}t & 0 \leq t < 1 \\ \frac{1}{2}t & -1 \leq t < 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \begin{cases} \frac{1}{2}t & 0 \leq t < 1 \\ -\frac{1}{2}t & -1 \leq t < 0 \\ 0 & \text{elsewhere} \end{cases}$$



6) Given the signal shown in figure below, write the mathematical equation for this signal:



$$\begin{aligned}
 x(t) = & \sin t [u(t-0) - u(t-2\pi)] \\
 & + 2 [u(t-7) - u(t-8)] \\
 & + \frac{2}{3}(t-9) [u(t-9) - u(t-10)] \\
 & + \frac{2}{3}(11-t) [u(t-10) - u(t-11)]
 \end{aligned}$$

VI- example

Homework

1.1. A continuous-time signal $x(t)$ is shown in Fig. 1-17. Sketch and label each of the following signals.

(a) $x(t - 2)$; (b) $x(2t)$; (c) $x(t/2)$; (d) $x(-t)$

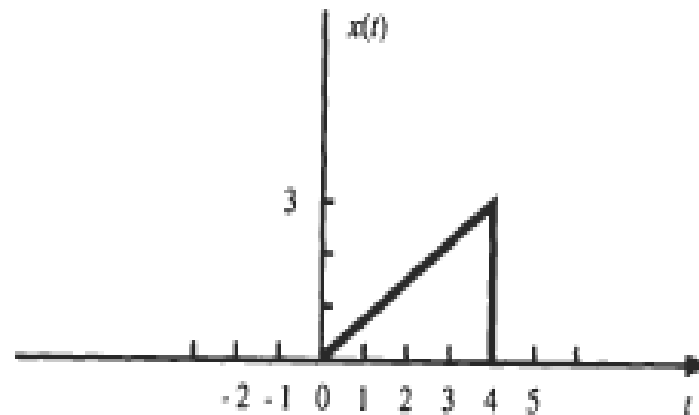


Fig. 1-17

1.2. A discrete-time signal $x[n]$ is shown in Fig. 1-19. Sketch and label each of the following signals.

(a) $x[n - 2]$; (b) $x[2n]$; (c) $x[-n]$; (d) $x[-n + 2]$

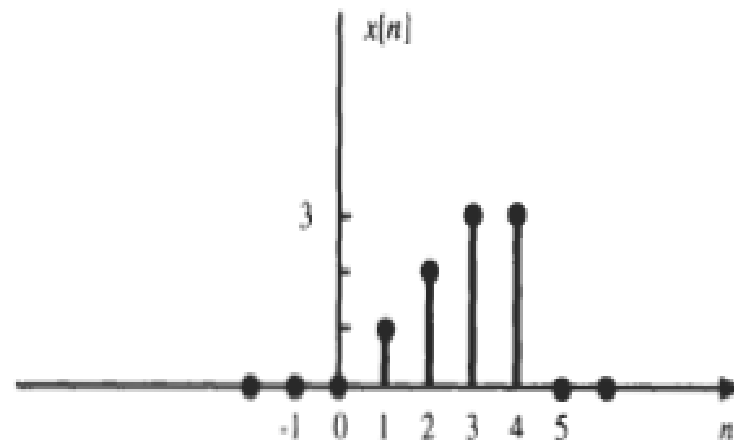


Fig. 1-19

1.4. Using the discrete-time signals $x_1[n]$ and $x_2[n]$ shown in Fig. 1-22, represent each of the following signals by a graph and by a sequence of numbers.

(a) $y_1[n] = x_1[n] + x_2[n]$; (b) $y_2[n] = 2x_1[n]$; (c) $y_3[n] = x_1[n]x_2[n]$

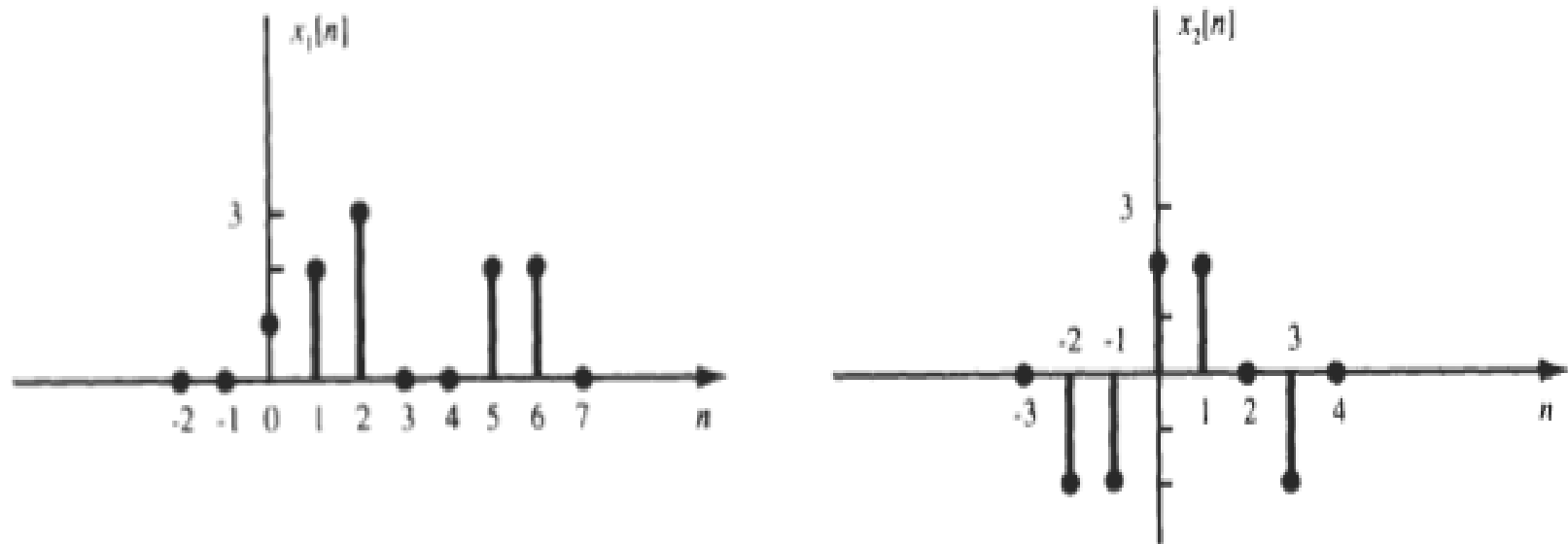


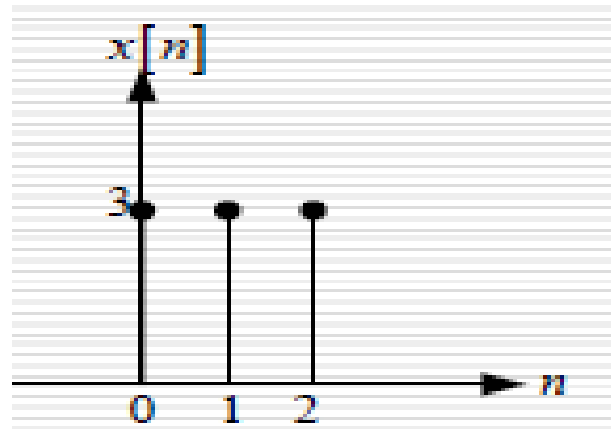
Fig. 1-22

- 1-5 Consider the following sinusoidal signal $x(t) = \cos 15t$
1- find ω_0, T_0, f_0
2- Find the fundamental period of $x[n] = x(nT_s)$ if $T_s = 0.1\pi$

1-6 Prove that

$$u(-t) = \begin{cases} 0 & t > 0 \\ 1 & t < 0 \end{cases}$$

- 1-7 The signal $x[n]$ is given below is it an energy or power signal.



1-8 Determine whether the signal given is an energy or power signal:

$$x(t) = e^{-at}u(t), \quad a > 0$$