S & S

Chapter 2

Introductions to Systems

Dr. Raghad Samir Al Najim

This chapter includes the following:

Definition of systems

Systems types

Classification of systems

Properties of systems

LTIS

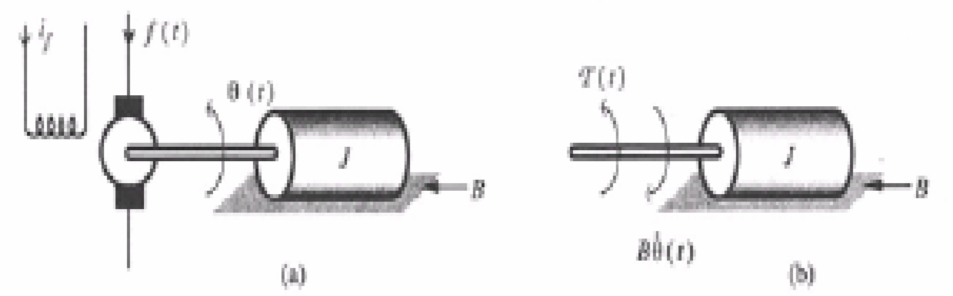
Systems output response

Definition of system

- The concept of *system* is useful in dealing with actual devices or processes for purposes of analysis and synthesis.
- A system operates on a signal (the input) to **modify, transform or re-express it in another form** (the output) which may be more desirable.
- A system is a mathematical model of a physical process that relates the input (or excitation) signal to the output (or response) signal.

SomeTypes of System

- There are many types of systems , some examples are :
- – Electrical, Mechanical, Fluid, Thermal
- – Biological, Chemical, Nuclear
- – Economic, Industrial,
- – Military,
- – Sports, ……

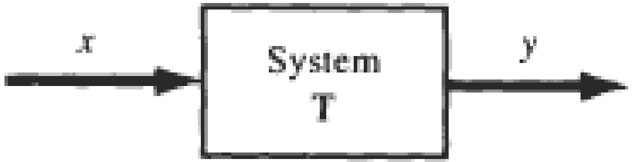


Classifications of Systems:

- **1. Continuous-time and discrete-time systems;**
- **2. Constant-parameter and time-varying-parameter systems; (note constant = time invariant)**
- **3. Instantaneous (memoryless) and dynamic (with memory) systems;**
- **4. Causal and non-causal systems;**
- **5. Lumped-parameter and distributed-parameter systems;**
- **6. Linear and nonlinear systems;**
- **7. Analog and Digital systems;**

How is a System Represented?

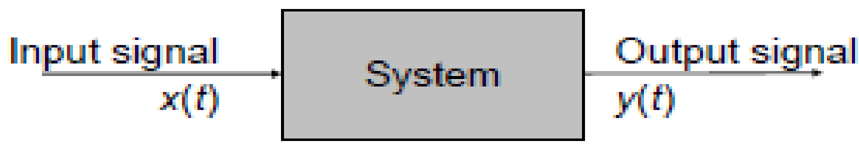
- Let x and y be the input and output signals, respectively, of a system. Then the system is viewed as a transformation (or mapping) of **x** into y. This transformation is represented by the mathematical notation $y = \mathbf{T}x$
- where **T** is the operator representing some well-defined **rule** by which **x** is transformed into y as shown

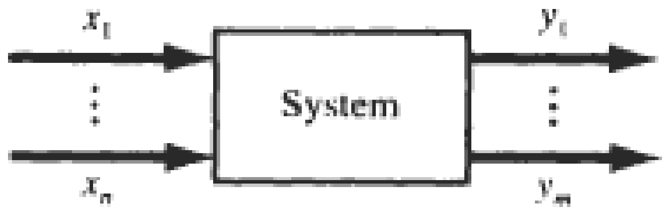


- system takes a signal as an input and transforms it into another signal (output)

A) System Represented

- A system can be represented as the ratio of the output signal over the input signal. That way, when we "multiply" the system by the input signal, we get the output signal.





B) Continuous Time and Discrete-Time Systems:

- If the input and output signals x and y are continuous-time signals, then the system is
- called a ***continuous-time system.***
- ***If the input and output signals are discrete-time*** signals or sequences, then the system is called a ***discrete-time system***

<u>C) Memoryless / Systems with Memory</u>

- A system is said to be ***memoryless*** *if the output at any time depends on only the input* at that same time. Otherwise, the system is said to have ***memory.***
- *An example of a* memoryless system is a resistor R with the input *x (t) taken as the current and the voltage* taken as the output *y (t) . The input-output relationship (Ohm's law) of a resistor is*

$$y(t) = R\,x(t) \quad ; \quad v(t) = R\,i(t)$$

An example of a system with memory is a capacitor ***C with the current as the input x(t)*** and the voltage as the output ***y (t) ; then***

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)\,d\tau$$

D) Causal and Non causal Systems:

- A system is ***causal*** *if its output y(t) at an arbitrary time t = to depends on only* the input *x(t) for t ≤to. That is, the output of a causal system at the present time depends* on **only the present and/or past values of the input**, not on its future values.
- Thus, in a causal system, it is not possible to obtain an output before an input is applied to the system. **A system is called *noncausal if it is not causal. As an example:***
- ***Y(t)=y(t-1)+x(t)+2 x(t-1)*** ***is causal***
- ***Y(t)=y(t+1)+x(t+1)-2 x(t-1)*** ***is NOT causal***

Note that all memoryless systems are causal, but not vice versa.

E) Linear Systems and Nonlinear Systems:

- If the operator T satisfies the following two conditions, then T is called a linear operator and the system represented by a linear operator T is called a linear system:
- ***1. Additivity:*** Given that Tx1 = y1, and Tx2 = y2, then **T{x1 +x2) =y1 +y2**
- 2. ***Homogeneity*** ***(or Scaling):*** for any signals x,any scalar α **T{αx}=α y**
- Any system that does not satisfy above equations, then it is classified as a nonlinear system. The above two equations can be combined into a single condition as:

$$T\{\alpha_1 x_1 + \alpha_2 x_2\} = \alpha_1 y_1 + \alpha_2 y_2$$

- where a, and a, are arbitrary scalars. This equation is known as the **superposition property.**
- Examples of linear systems are :differentiation, Integration,

$$(d\, y / d\, t) + 3\, y(t) = f(t)$$

- Examples of nonlinear systems are: Human Hearing, Human Vision, $y = x^2$

$$y = \cos x$$

F) Time-Invariant and Time-Varying Systems:

- A system is called time-invariant if a time shift (delay or advance) in the input signal causes the same time shift in the output signal. Thus, for a **continuous- Discrete-time system, the system is time-invariant** if for any real value of τ,k.

$$\mathbf{T}\{x(t-\tau)\} = y(t-\tau) \qquad \mathbf{T}\{x[n-k]\} = y[n-k]$$

- . *A system which does not satisfy above equations:* is called a **time-varying system.**

-
  - Time Varying Systems
    - – Amplifier with Time-Varying Gain $y(t) = \underline{t}x(t)$
    - – First-Order System $\dot{y}(t) + \underline{a(t)}y(t) = bx(t)$

- **Transcendental system**
- *Answer: Time-invariant* $y(t) = \cos(x(t))$
- **Squarer**
- *Answer: Time-invariant* $y(t) = x^2(t)$
- **Differentiator** $y(t) = \frac{d}{dt}x(t)$
- *Answer: Time-invariant* $\frac{d}{dt}x(t-\tau) = y(t-\tau)$

Check if the given discrete time system is Time Invariant??

$$y[n] = \frac{1}{2}(x[n] + x[n-1])$$

$$T\{x[n-n_d]\} = \frac{1}{2}(x[n-n_d] + x[n--1-n_d])$$

$$y[n-n_d] = \frac{1}{2}(x[n-n_d] + x[n-n_d-1])$$

Result when delayed at the inputs is similar when the delayed at the outputs.
y[n] is time-invariant

Tips

To test $T\{x[n-n_d]\}$, minus whatever inside the input signal bracket with n_d.

To test $y[n-n_d]$, change all n with $n-n_d$.

G) Linear Time-Invariant Systems

- If the system is linear, time-invariant and causel, then it is called a linear Time-invariant (LTI) system.:
- 1- Continous: **LTICT system**
- 2- Discrete: **LTIDT system**

H) Stable & Unstable Systems:

- A system is ***bounded-input/bounded-output (BIBO) stable if for any bounded input x***
- defined by $|x| \le k_1$
- the corresponding output y is also bounded and defined by $|y| \le k_2$
- where ***k1 , and k2, are finite real constants.***

Example1:

- Table (1),” Examples of CT and DT systems with and without memory”

Continuous-time		Discrete-time	
Memoryless systems	Systems with memory	Memoryless systems	Systems with memory
$y(t) = 3x(t) + 5$	$y(t) = x(t-5)$	$y[k] = 3x[k] + 7$	$y[k] = x[k-5]$
$y(t) = \sin\{x(t)\} + 5$	$y(t) = x(t+2)$	$y[k] = \sin(x[k]) + 3$	$y[k] = x[k+3]$
$y(t) = e^{x(t)}$	$y(t) = x(2t)$	$y[k] = e^{x[k]}$	$y[k] = x[2k]$
$y(t) = x^2(t)$	$y(t) = x(t/2)$	$y[k] = x^2[k]$	$y[k] = x[k/2]$

Example 2: on causal Systems

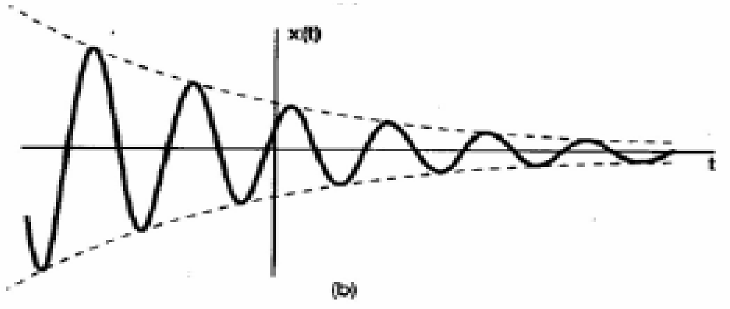
- Table(2). Examples of causal and non-causal systems

CT systems		DT systems	
Causal	Non-causal	Causal	Non-causal
$y(t) = x(t-5)$	$y(t) = x(t+2)$	$y[k] = 3x[k-1]+7$	$y[k] = x[k+3]$
$y(t) = \sin\{x(t-4)\}+3$	$y(t) = \sin\{x(t+4)\}+3$	$y[k] = \sin(x[k-4])+3$	$y[k] = \sin(x[k+4])+3$
$y(t) = e^{x(t-2)}$	$y(t) = x(2t)$	$y[k] = e^{x[k-2]}$	$y[k] = x[2k]$
$y(t) = x^2(t-2)$	$y(t) = x(t/2)$	$y[k] = x^2[k-5]$	$y[k] = x[k/2]$
$y(t) = x(t-2)+x(t-5)$	$y(t) = x(t-2)+x(t+2)$	$y[k] = x[k-2]+x[k-8]$	$y[k] = x[k+2]+x[k-8]$

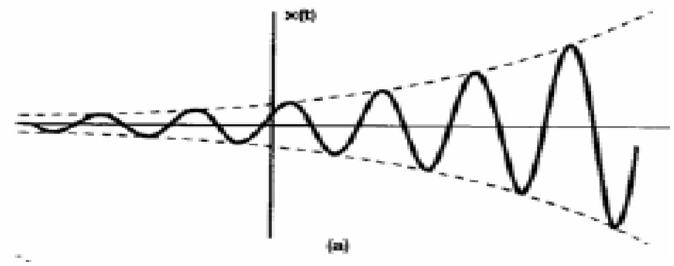
Example 3 : Check if the given system are stable.

- $y[k] = 50 \sin(x[k]) + 10$.
- Note that $\sin(x[k])$ is bounded between $[-1, 1]$ for any arbitrary choice of $x[k]$. The output $y[k]$ is therefore bounded within the interval $[-40, 60]$. Therefore this system is always stable.

Stable



Unstable



Response of LTI system to a Step and Impulse Inputs

Mathematical Models of Systems

- ***Continuous-Time Systems:*** continuous signals are transformed via **differential equations**.
- E.g. Electric circuits, car velocity

$$\frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}v_s(t)$$

$$m\frac{dv(t)}{dt} + \rho v(t) = f(t)$$

First order differential equations

- ***Discrete-Time Systems:*** discrete signals are transformed via **difference equations**
- E.g. bank account, discrete car

$$y[n] = 1.01y[n-1] + x[n]$$

$$v[n] - \frac{m}{m+\rho\Delta}v[n-1] = \frac{\Delta}{m+\rho\Delta}f[n]$$

$$\frac{dv(n\Delta)}{dt} = \frac{v(n\Delta) - v((n-1)\Delta)}{\Delta}$$

First order difference equations

Response of LTIC system to a Step and Impulse inputs

For LTICT system , this system is represented using differential equation, so to solve such system to a special inputs like(Step and Impulse), then one can solve such problem

either using differential equations ,

or **the Laplce transform.**

Laplace Transform and Transfer function:

TRANSFER FUNCTIONS

The models of systems are often written in the form of a ratio of Output/Input. If the models are turned into a function of s it is called a transfer function and this is usually denoted as G(s).

$$G(s) = \frac{\text{Output}}{\text{Input}} \quad \text{The output and input are functions of s.}$$

$$\frac{d\theta}{dt} \text{ becomes } s\,\theta \qquad \frac{d^2\theta}{dt^2} \text{ becomes } s^2\,\theta \qquad \frac{d^n\theta}{dt^n} \text{ becomes } s^n\,\theta$$

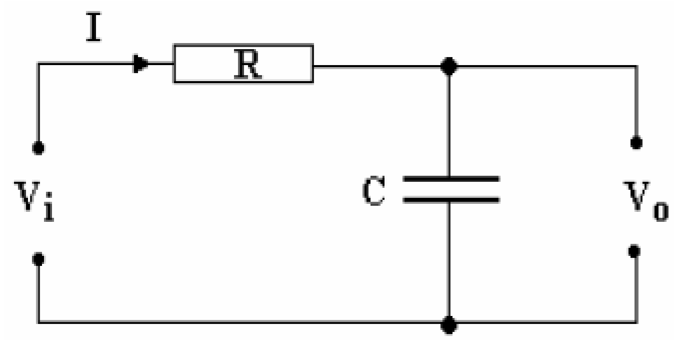
$$Transfer\ function\ (T.F) = G(s) = \left.\frac{\mathcal{L}[output]}{\mathcal{L}[input]}\right|_{zero\ initial\ conditions}$$

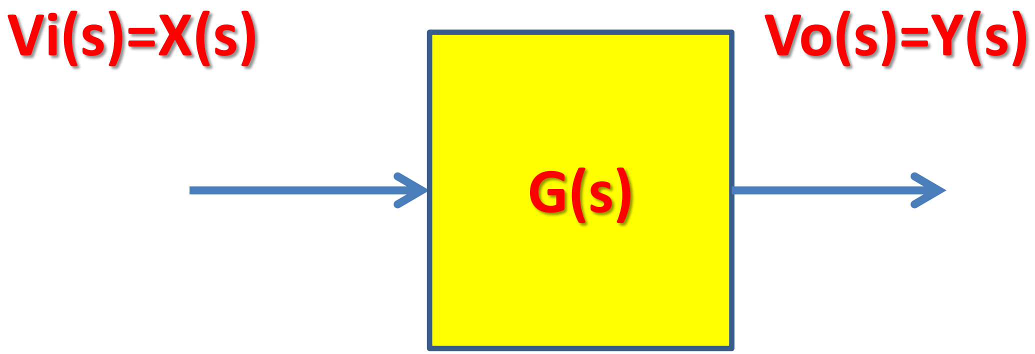
$$T.F = G(s) = \frac{b_m\,s^m + b_{m-1}\,s^{m-1} + \cdots + b_1\,s + b_o}{s^n + a_{n-1}\,s^{n-1} + \cdots + a_1\,s + a_o} \qquad ; \quad (n \geq m)$$

Example of an electrical system:

R -C SERIES CIRCUIT

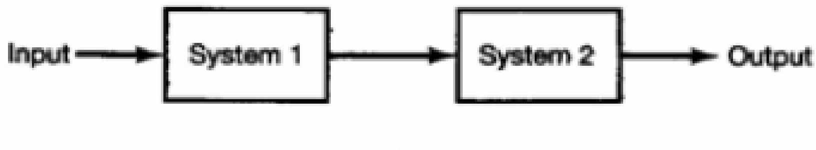
$$G(s) = \frac{V_o}{V_i}(s) = \frac{1}{Ts + 1}$$



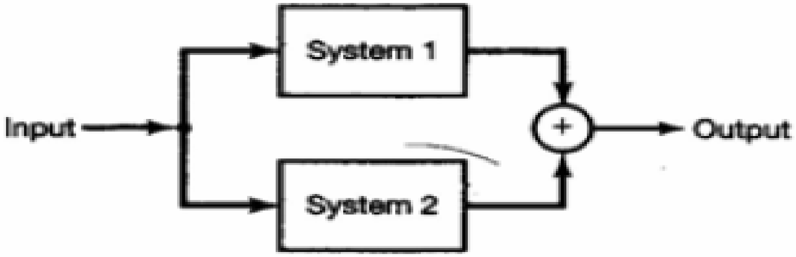


I) Systems Interconnections

- Useful for controls , filters and sensor fusion, there are many ways to connect systems such as:



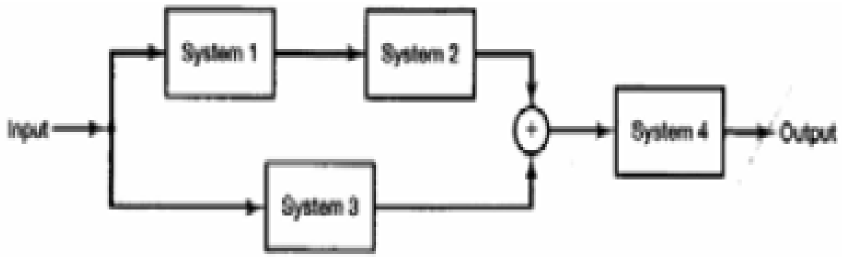
(a)

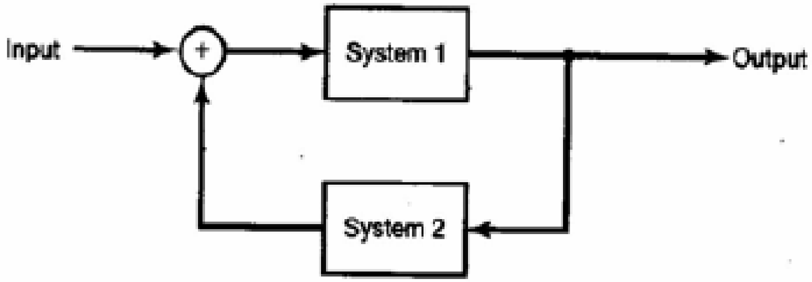


(b)

- Series

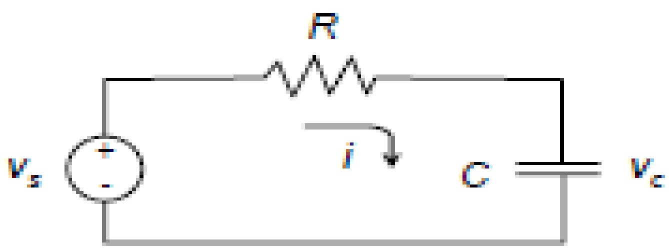
Parallel





- Combination

Feedback

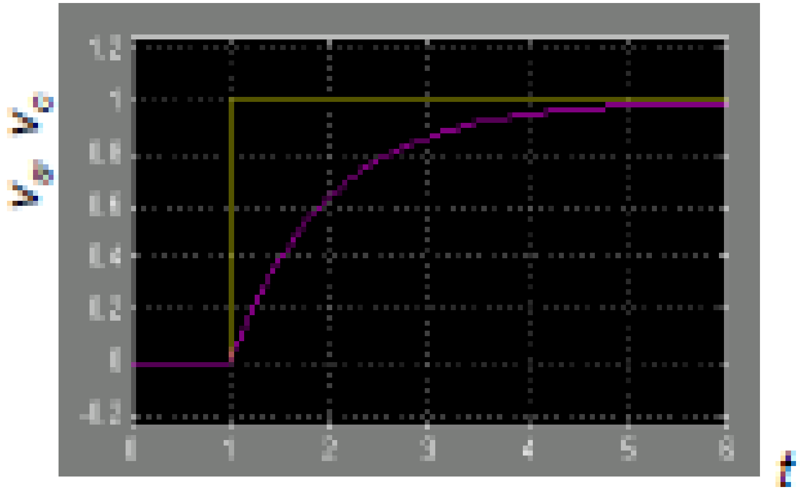
- **Example 4:** Signals in an Electrical Circuit(RC-circuit). The signals *vc* and *vs* are patterns of variation over time
-
- 

$$i(t) = \frac{v_s(t) - v_c(t)}{R}$$

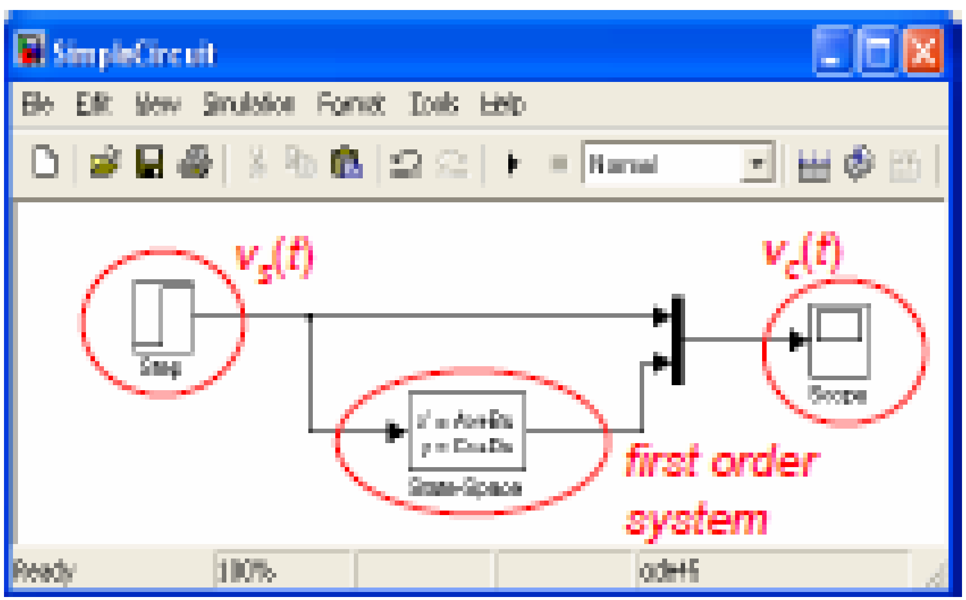
$$i(t) = C\frac{dv_c(t)}{dt}$$

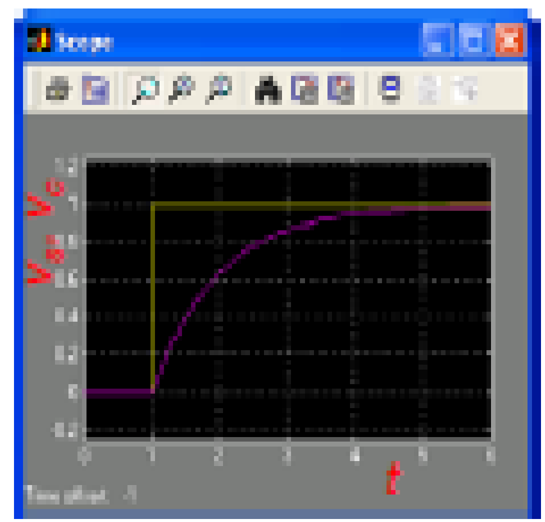
$$\frac{dv_c(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}v_s(t)$$

$$\frac{v_c(s)}{v_s(s)} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}}$$



- Step (signal) v_s at t=1
- RC = 1
- First order (exponential) response for v_c

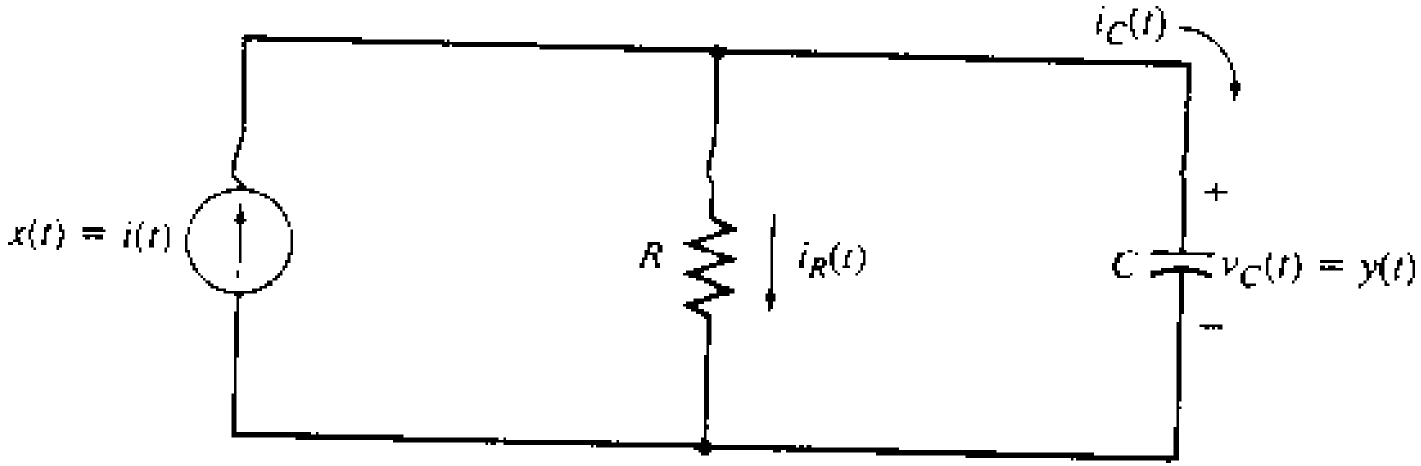




Example 5: Find the output response (o/p y(t)) for the following circuit. If the input x(t) is :

1- unit step

2- Impulse



First of all, find the Transfer function that relates input X(s) and the output Y(s)

$$x(t) = i(t)\,;\; y(t) = V_C$$

$$i(t) = i_R(t) + i_C(t)$$

$$C\frac{dy}{dt} + \frac{1}{R}y(t) = x(t)$$

Taking the Laplace transform for last equation, we have

$$C\,sY(s) + \frac{1}{R}Y(s) = X(s)$$

$$\text{the Transfer Function}\ \frac{Y(s)}{X(s)} = \frac{R}{RC\,s+1}$$

If the input x(t) is u(t) (Unit step), Taking the Laplace transform for u(t)=1/s, and substitute in the T.F, we get

$$Y(s)=\frac{R}{RC\,s+1}\,X(s)\quad ;\quad Y(s)=\frac{R}{RC\,s+1}\,\frac{1}{s}$$

$$Y(s)=\frac{\frac{1}{C}}{s(s+\frac{1}{RC})}=\frac{A}{s}+\frac{B}{s+\frac{1}{RC}}$$

$$A=\left.\frac{\frac{1}{C}}{(s+\frac{1}{RC})}\right|_{s=0}=R\ ;\ B=\left.\frac{\frac{1}{C}}{s}\right|_{s=-\frac{1}{RC}}=-R$$

$$\therefore Y(s)=\frac{R}{s}-\frac{R}{s+\frac{1}{RC}}\ ;\ \ y(t)=l^{-1}\big(Y(s)\big)$$

$$\therefore\ y(t)=R-R\,e^{-\frac{1}{RC}t}=R(1-e^{-\frac{1}{RC}t})$$

$For\ R\ and\ C=1,\ then\ we\ have$

$$y(t)=1-e^{-t}$$

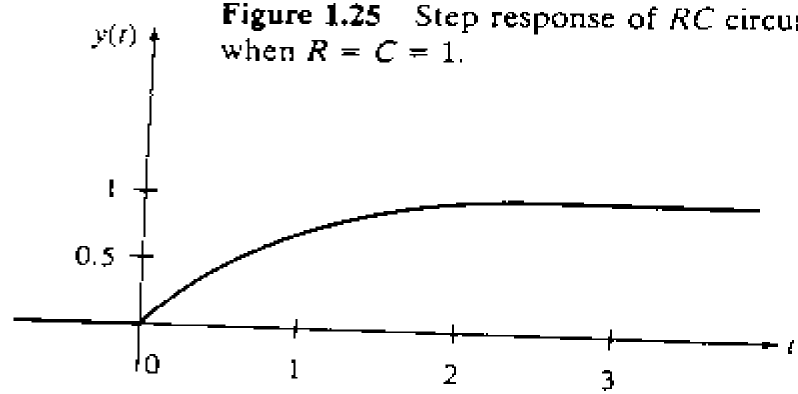


Figure 1.25 Step response of RC circuit when $R = C = 1$.

If the input x(t) is δ(t) (Impulse signal), Taking the Laplace transform for δ(t)=1, then substitute in the T.F, we get

$$Y(s) = \frac{R}{RC\,s+1}\,X(s) \quad ; \quad Y(s) = \frac{R}{RC\,s+1}$$

$$Y(s) = \frac{\frac{1}{C}}{(s+\frac{1}{RC})} \quad ; \quad y(t) = l^{-1}\big(Y(s)\big)$$

$$\therefore\ y(t) = \frac{1}{C}e^{-\frac{1}{RC}t} \quad ;\ For\ R\ and\ C = 1, then\ we\ have$$

$$y(t) = e^{-t}$$

Response of LTID system to a step & impulse inputs

For LTIDT system , this system is represented using difference equation, so to solve such system to a special inputs like(Step and Impulse), then one can solve such problem either using difference equations , or the Z- transform(both of them wil be considered in DSP course).

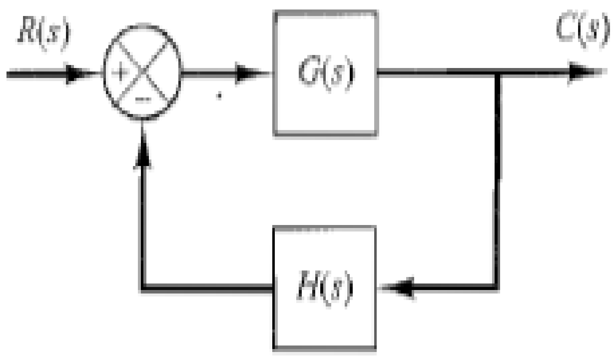
***Example 6:* Consider the system shown in the block diagram,**

1- find the T.F =C(s)/ R(s) .

2- Find y(t) for a step input (homework).

G(s)= 1/ s+2 ;
H(s)= 1

R(s) is input,
C(s) = output,
G(s) is system,
H(s) is the feedback system



Feedback closed loop CLTI system

Given that G(s)= 1/ s+2 ; H(s)= 1

First step is to find the Transfer function of the overall system that relates C(s) to R(s) using the given block diagram:

C(s)= G(s) {R(s)-H(s) C(s)}

C(s)= G(s) R(s) – G(s) H(s) C(s)

C(s){ 1+G(s) H(s) }= G(s) R(s)

Then T.F C(s)/ R(s) is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\,H(s)}$$

Substitute for the given G(s),H(s)

$$\frac{C(s)}{R(s)} = \frac{\frac{1}{s+2}}{1+\frac{1}{s+2}*1} = \frac{\frac{1}{s+2}}{\frac{s+2+1}{s+2}}$$

$$\frac{C(s)}{R(s)} = \frac{1}{s+3}$$