Chapter Four

Fourier Transform of CT signals & Systems

(Frequency Response)

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Introduction

There are two ways to analyze and design CT systems:
- The Fourier Transform (used in signal processing)
-The Laplace Transform (used in linear control systems)

The Fourier Transform is a particular case of the Laplace Transform, so the properties of Laplace transforms are inherited by Fourier transforms. One can compute Fourier transforms in the same way as Laplace transforms.

Relationship with Laplace transform

The Fourier Transform is a particular case of Laplace transform

$$\boxed{L(s) = \int_{-\infty}^{\infty} g(t) e^{-st} dt}$$

$$\boxed{F(j\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt}$$

$$\boxed{F(j\omega) = L(j\omega)}$$

$$\boxed{S=jw}$$

$F(j\omega)$ is often called spectrum or amplitude spectral density (spectral refers to 'variation with respect to frequency', density refers to 'amplitude per unit frequency')

Connection between the Fourier Transform and the Laplace Transform:

This equation defines the Fourier transform of x(t):

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

The Laplace transform of x(t), is given by

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

Comparing these Eqs. , we see that the Fourier transform is a special case of the Laplace transform in which s = jω, that is,

$$X(s)\big|_{s=j\omega} = \mathscr{F}\{x(t)\}$$

Fourier Transform Pairs

For any signal f(t) , FT and IFT are given by

Define

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega$$

and we say that f(t) and F(w) form a Fourier transform pair denoted by f(t)↔ F(w)

Some FT Notation:

If $X(\omega)$ is the Fourier transform of $x(t)$…

then we can write this in several ways:

1. $x(t) \leftrightarrow X(\omega)$

2. $X(\omega) = \mathfrak{F}\{x(t)\}$ $\Rightarrow$ $\mathfrak{F}\{\ \}$ is an "operator" that operates on $x(t)$ to give $X(\omega)$

3. $x(t) = \mathfrak{F}^{-1}\{X(\omega)\}$ $\Rightarrow$ $\mathfrak{F}^{-1}\{\ \}$ is an "operator" that operates on $X(\omega)$ to give $x(t)$

PROPERTIES OF THE CONTINUOUS-TIME FOURIER TRANSFORM

A. Linearity:

$$a_1 x_1(t) + a_2 x_2(t) \leftrightarrow a_1 X_1(\omega) + a_2 X_2(\omega)$$

B. Time Shifting:

$$x(t - t_0) \leftrightarrow e^{-j\omega t_0} X(\omega)$$

C. Frequency Shifting:

$$e^{j\omega_0 t} x(t) \leftrightarrow X(\omega - \omega_0)$$

D. Time Scaling:

$$x(at) \leftrightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$$

E. Time Reversal:

$$x(-t) \leftrightarrow X(-\omega)$$

F. Duality (or Symmetry):

$$X(t) \leftrightarrow 2\pi x(-\omega)$$

Besides the inverse relationship of frequency and time, by interchanging the frequency and the time variables in the definitions of the direct and the inverse Fourier transform similar equations are obtained. Thus, the direct and the inverse Fourier transforms are dual. *This duality property allows us to obtain the Fourier transform of signals for which we already have a Fourier pair and that would be difficult to obtain directly. It is thus one more method to obtain the Fourier transform, besides the Laplace transform and the integral definition of the Fourier transform.*

G. Differentiation in the Time Domain:

$$\frac{dx(t)}{dt} \leftrightarrow j\omega X(\omega)$$

H. Differentiation in the Frequency Domain:

$$(-jt)x(t) \leftrightarrow \frac{dX(\omega)}{d\omega}$$

I. Integration in the Time Domain:

$$\int_{-\infty}^{t} x(\tau)\, d\tau \longleftrightarrow \pi X(0)\, \delta(\omega) + \frac{1}{j\omega} X(\omega)$$

J. Convolution:

$$x_1(t) * x_2(t) \longleftrightarrow X_1(\omega) X_2(\omega)$$

Table (1): Summery of FT properties

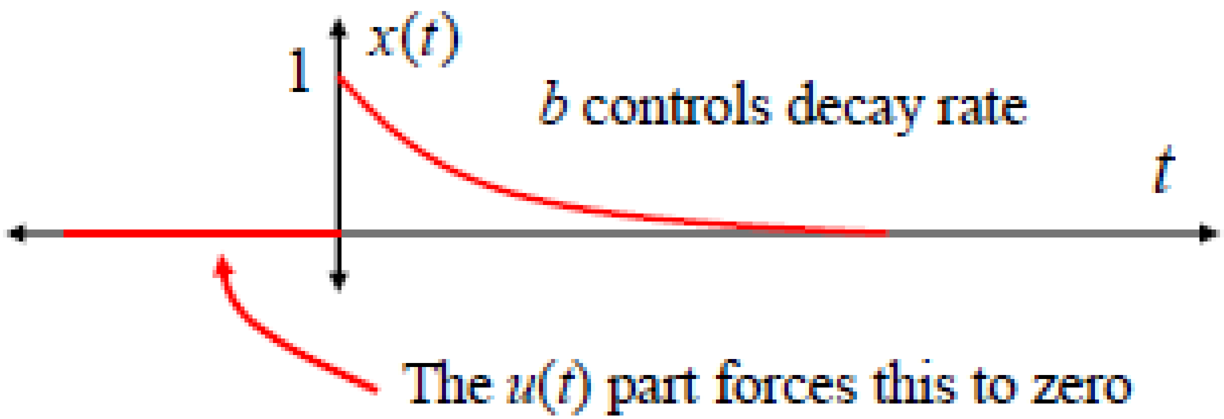
Property	Signal	Fourier transform
	$x(t)$	$X(\omega)$
	$x_1(t)$	$X_1(\omega)$
	$x_2(t)$	$X_2(\omega)$
Linearity	$a_1 x_1(t) + a_2 x_2(t)$	$a_1 X_1(\omega) + a_2 X_2(\omega)$
Time shifting	$x(t - t_0)$	$e^{-j\omega t_0} X(\omega)$
Frequency shifting	$e^{j\omega_0 t} x(t)$	$X(\omega - \omega_0)$
Time scaling	$x(at)$	$\frac{1}{\lvert a \rvert} X\left(\frac{\omega}{a}\right)$
Time reversal	$x(-t)$	$X(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Time differentiation	$\frac{dx(t)}{dt}$	$j\omega X(\omega)$
Frequency differentiation	$(-jt)x(t)$	$\frac{dX(\omega)}{d\omega}$
Integration	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\pi X(0)\delta(\omega) + \frac{1}{j\omega} X(\omega)$
Convolution	$x_1(t) * x_2(t)$	$X_1(\omega) X_2(\omega)$

Examples of FT of some signals

Example : Find the FT for the decaying exponential signal

Given a signal $x(t) = e^{-bt}u(t)$ find $X(\omega)$ if $b > 0$

<u>**Solution**</u>: First see what $x(t)$ looks like:



$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$X(\omega) = \int_{-\infty}^{\infty} \underbrace{e^{-bt} u(t) e^{-j\omega t}}_{\substack{\text{integrand} = 0 \text{ for } t < 0 \\ \text{due to the } u(t)}} dt = \int_{0}^{\infty} e^{-bt} e^{-j\omega t} dt = \int_{0}^{\infty} e^{-(b+j\omega)t} dt$$

Set lower limit to 0 and then $u(t) = 1$ over integration range

Easy integral!

$$= \left[\frac{-1}{b+j\omega} e^{-(b+j\omega)t} \right]_{t=0}^{t=\infty} = \frac{-1}{b+j\omega} \left[e^{-(b+j\omega)\infty} - e^{-(b+j\omega)0} \right]$$

$$= \frac{1}{b+j\omega}$$

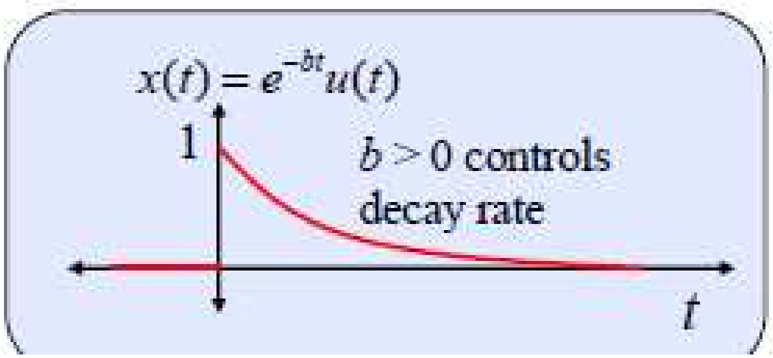
$$\boxed{x(t) = e^{-bt}u(t)} \longleftrightarrow \boxed{X(\omega) = \frac{1}{b + j\omega}}$$

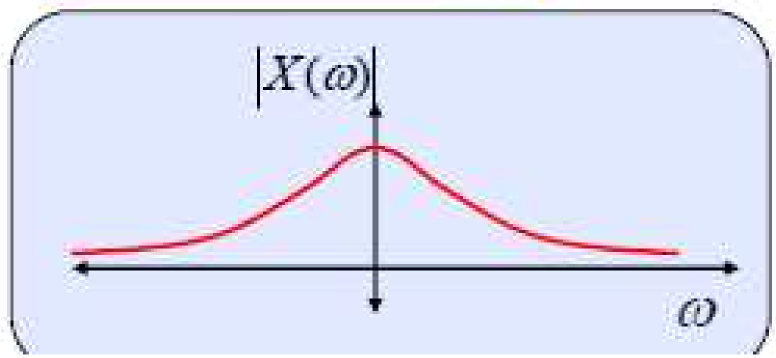
For $b > 0$

(Complex Valued)

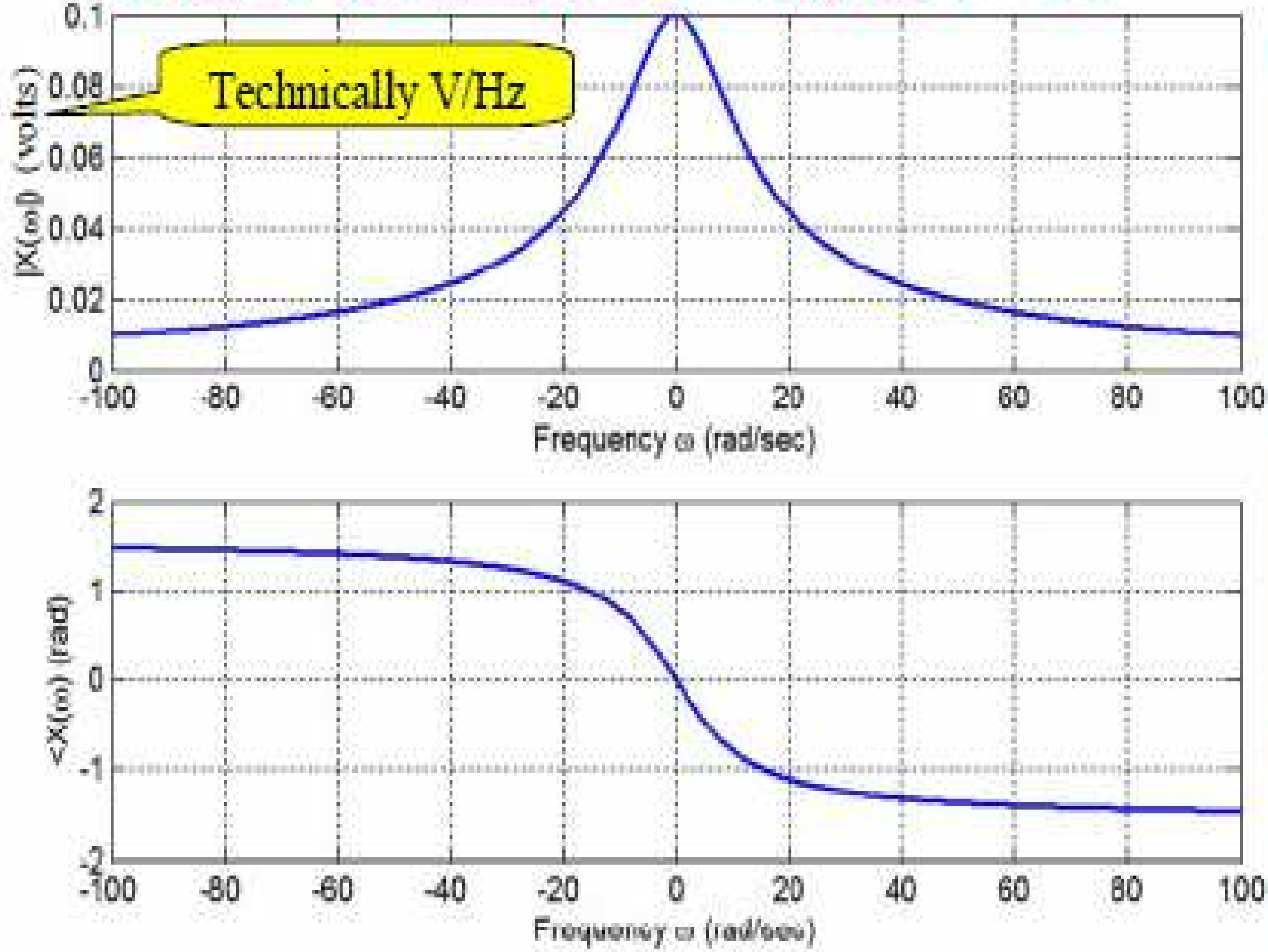
$$\left|X(\omega)\right| = \frac{1}{\sqrt{b^2 + \omega^2}} \quad \text{Magnitude}$$

$$\angle X(\omega) = -\tan^{-1}\left(\frac{\omega}{b}\right) \quad \text{Phase}$$





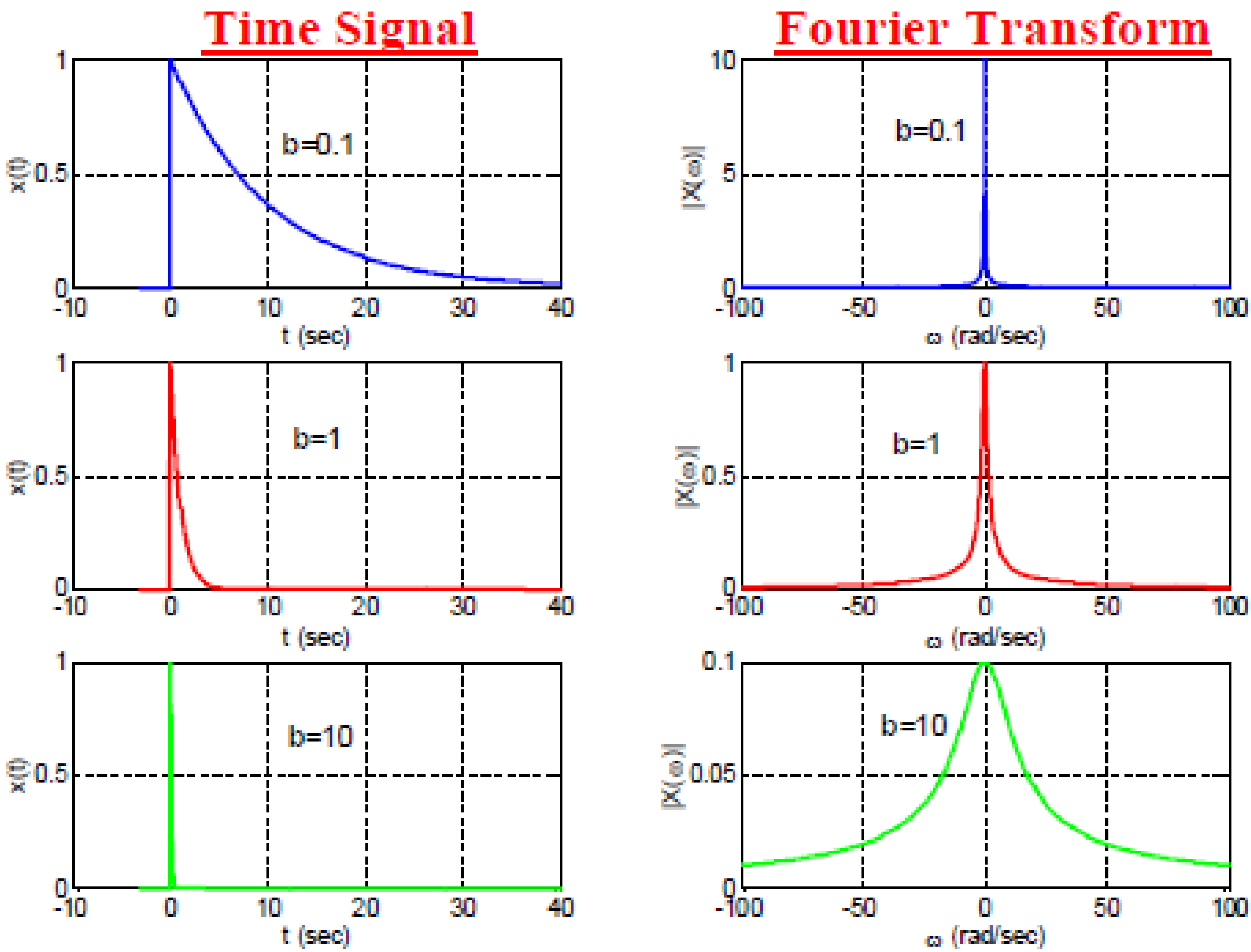
Fourier Transform of $e^{-bt}u(t)$ for $b = 10$



Note that magnitude plot has <u>even</u> symmetry

Note that phase plot has <u>odd</u> symmetry

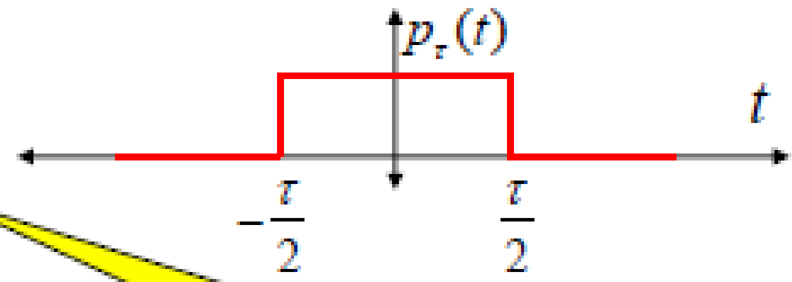
True for <u>every</u> real-valued signal


Time Signal
Fourier Transform
b=0.1
b=1
b=10
x(t)
t (sec)
|X(ω)|
ω (rad/sec)

Example: FT of a Rectangular pulse

τ = pulse width

Given: a rectangular pulse signal $p_\tau(t)$



Recall: we use this symbol to indicate a rectangular pulse with width τ

Find: $P_\tau(\omega)$… the FT of $p_\tau(t)$

Note the Notational Convention: lower-case for time signal and corresponding upper-case for its FT

Solution:

Note that

$$p_\tau(t) = \begin{cases} 1, & -\frac{\tau}{2} \le t \le \frac{\tau}{2} \\ 0, & otherwise \end{cases}$$

Now apply the definition of the FT:

$$P_\tau(\omega) = \int_{-\infty}^{\infty} p_\tau(t) e^{-j\omega t} dt = \int_{-\tau/2}^{\tau/2} e^{-j\omega t} dt$$

Limit integral to where $p_\tau(t)$ is non-zero… and use the fact that it is 1 over that region

$$= \frac{-1}{j\omega}\left[e^{-j\omega t}\right]_{\frac{-\tau}{2}}^{\frac{\tau}{2}} = \frac{2}{\omega}\underbrace{\left[\frac{e^{j\frac{\omega\tau}{2}} - e^{-j\frac{\omega\tau}{2}}}{j2}\right]}_{= \sin\left(\frac{\omega\tau}{2}\right)}$$

Artificially inserted 2 in numerator and denominator

Use Euler's Formula

$$\boxed{P_\tau(\omega) = \frac{2\sin\left(\frac{\omega\tau}{2}\right)}{\omega}}$$

sin goes up and down between -1 and 1

$1/\omega$ decays down as $|\omega|$ gets big… this causes the overall function to decay down

For τ=1/2

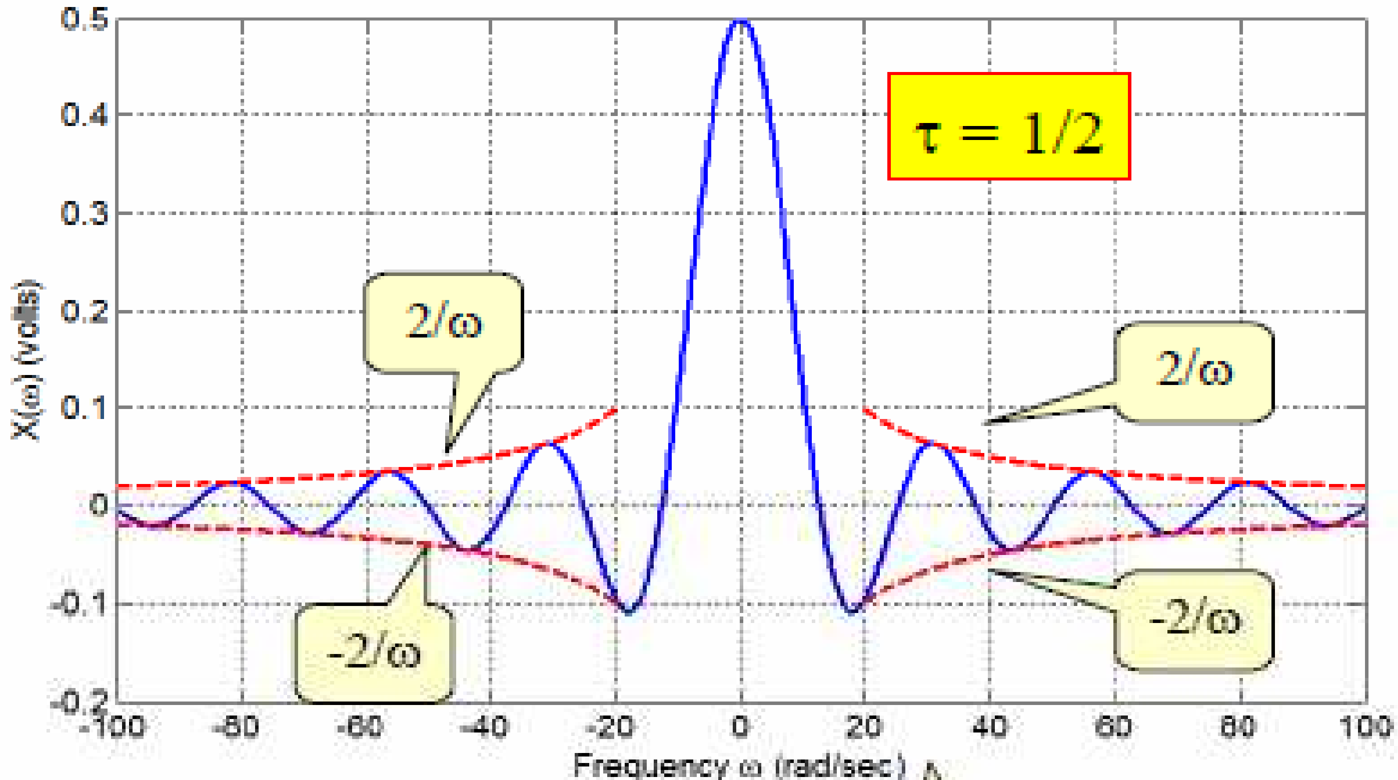
w	0	10	20	30	40	50
P(w)	0.5	0.119	-0.09	0.06	-0.02	------

$$note : to\ evaluate\ p(0),we\ can\ use\ the\ proerty\ of$$

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1$$

$$p(w)\Big|_{w=0}=\frac{2\sin\frac{w}{4}}{w}=\frac{2\sin\frac{w}{4}}{4\frac{w}{4}}=\frac{1}{2}\lim_{w\to 0}\left(\frac{\sin\frac{w}{4}}{\frac{w}{4}}\right)=\frac{1}{2}$$

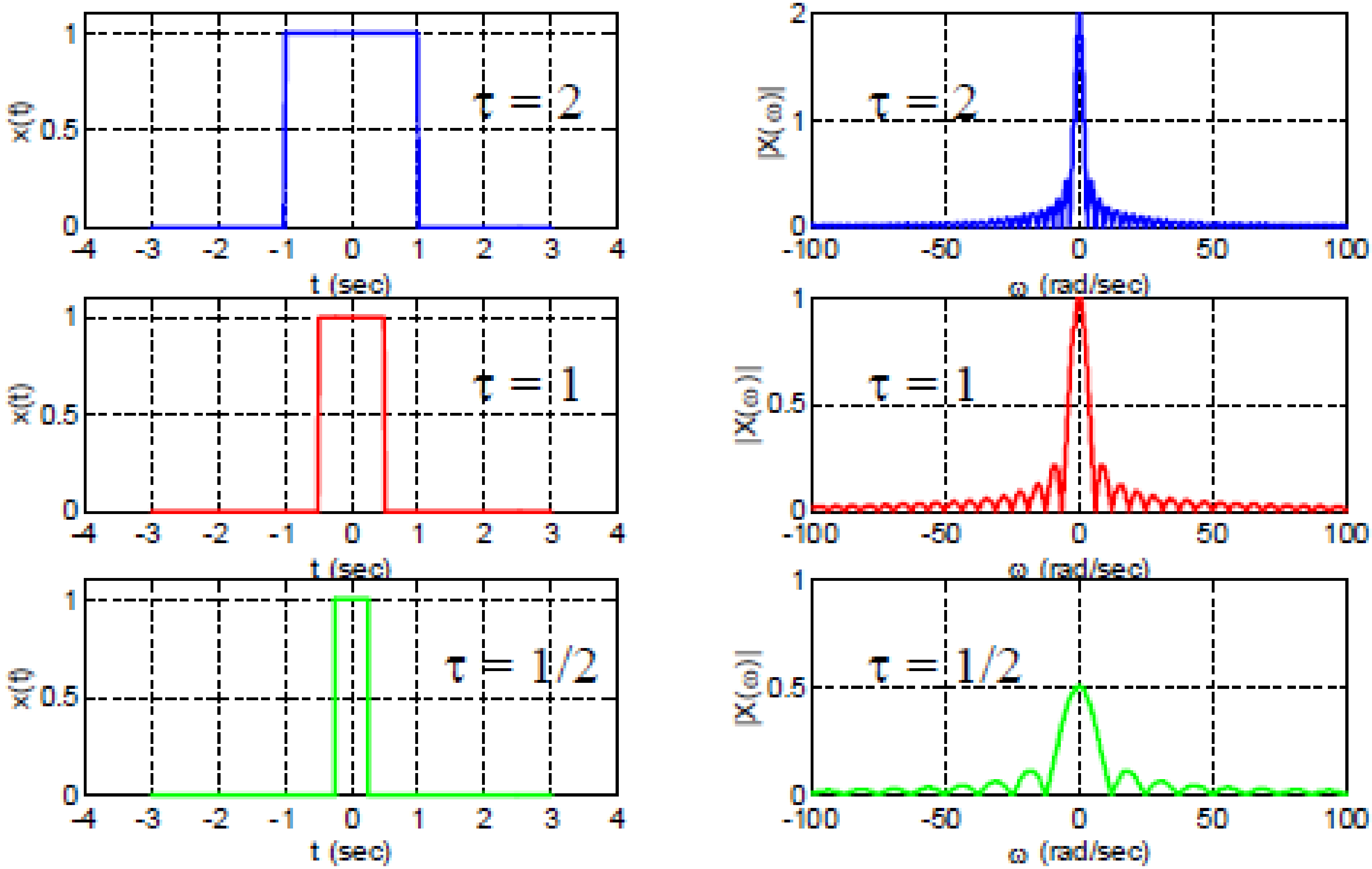
For ***this*** case the FT is real valued so we can plot it using a single plot (shown in solid blue here):



$$P_\tau(\omega) = \frac{2\sin\left(\frac{\omega\tau}{2}\right)}{\omega}$$

The sine wiggles up & down "between ±2/ω"

Effect of Pulse Width on the FT $P_\tau(\omega)$



Example: Find FT for δ(t)

$$\mathfrak{F}\{\delta(t)\} = \underbrace{\int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = e^{-j\omega 0}}_{\boxed{\text{Sifting property}}} = 1 \quad \Rightarrow \quad \boxed{\delta(t) \leftrightarrow 1}$$

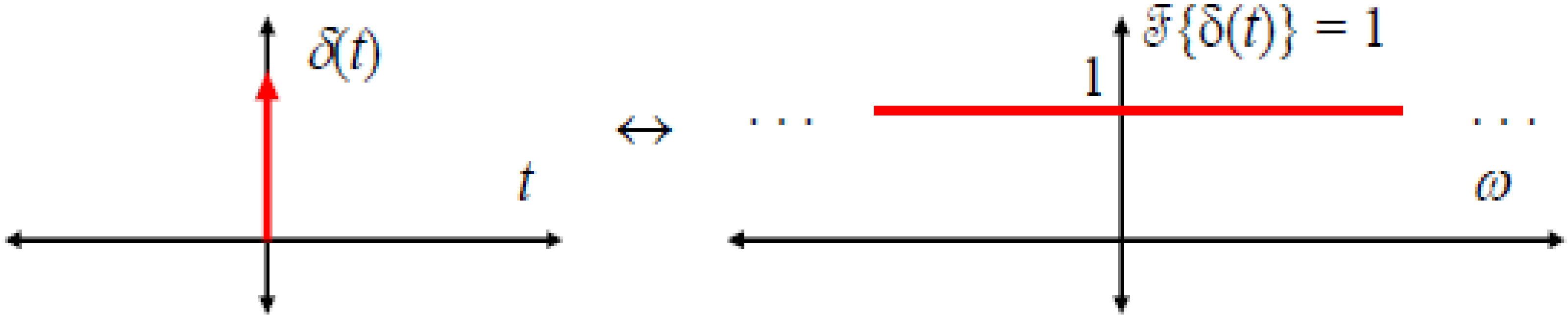


Table (2): Some examples for FT pairs

$$1 \leftrightarrow 2\pi\delta(\omega)$$

$$u(t) \leftrightarrow \pi\delta(\omega) + \frac{1}{j\omega}$$

$$e^{-at}u(t) \leftrightarrow \frac{1}{j\omega + a}$$

$$\cos(at) \leftrightarrow \pi(\delta(\omega - a) + \delta(\omega + a))$$

$$\sin(at) \leftrightarrow j\pi(\delta(\omega + a) - \delta(\omega - a))$$

$$rect(t) \leftrightarrow \text{sinc}(\omega / 2\pi)$$

$$\text{sinc}(t) \leftrightarrow rect(\omega / 2\pi)$$

Time Domain and Frequency Domain Response of LTICT systems

Introduction

Input-Output Relationship Characterized Two Ways

1. Time-Domain: $y(t) = h(t) * f(t)$

2. Freq-Domain: $Y(\omega) = H(\omega)F(\omega)$

Given input $f(t)$ and impulse response $h(t)$, to analyze the system we could either:

1. Compute the convolution $h(t) * f(t)$

   or…

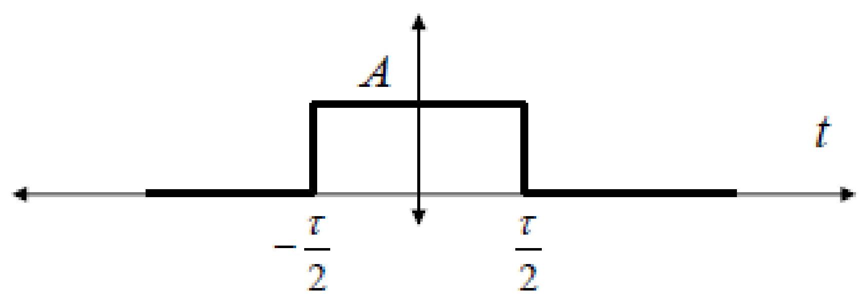
2. Do the following:
   - (a) Compute $H(\omega)$ & compute $F(\omega)$
   - (b) Compute the product $Y(\omega) = H(\omega)F(\omega)$
   - (c) Compute the IFT: $y(t) = \mathfrak{F}^{-1}\{ H(\omega)F(\omega)\}$

Example

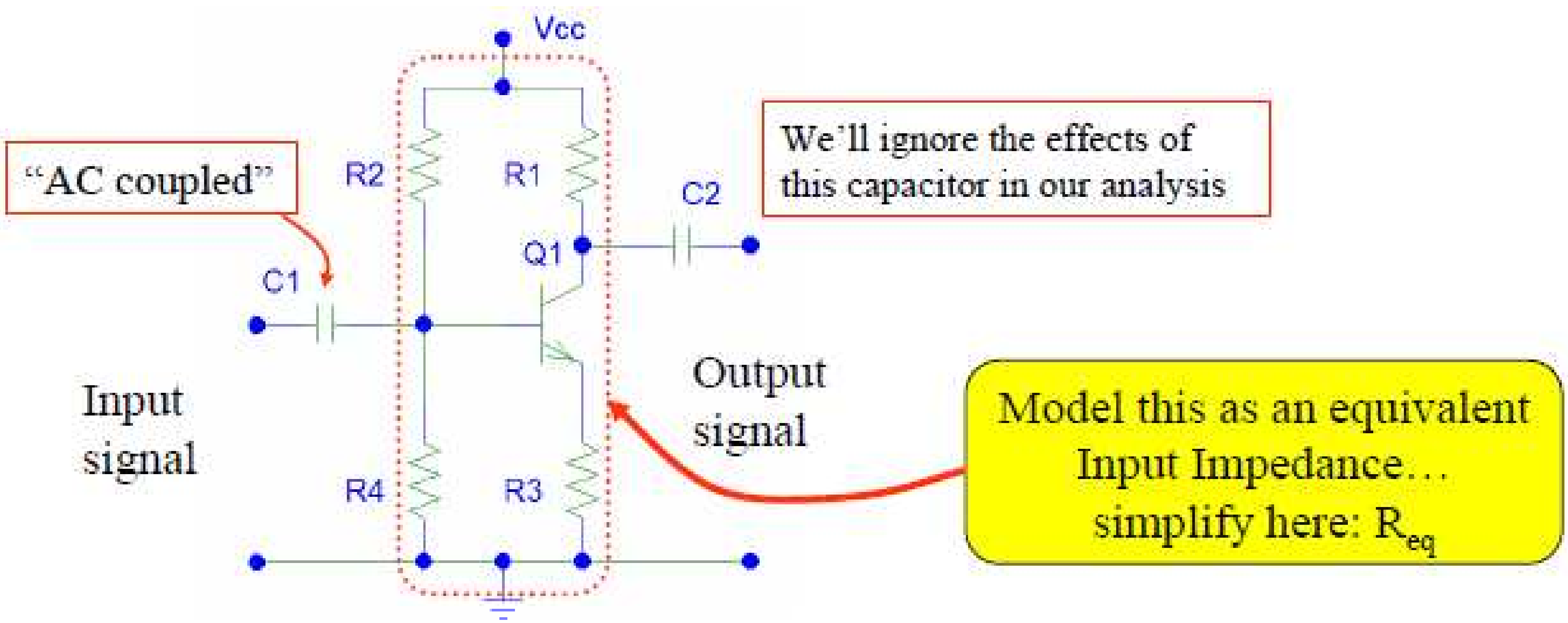
Suppose :You need to send a pulse signal into a computer's interface circuit to initiate an event , What kind of signal should you use?

A rectangular pulse

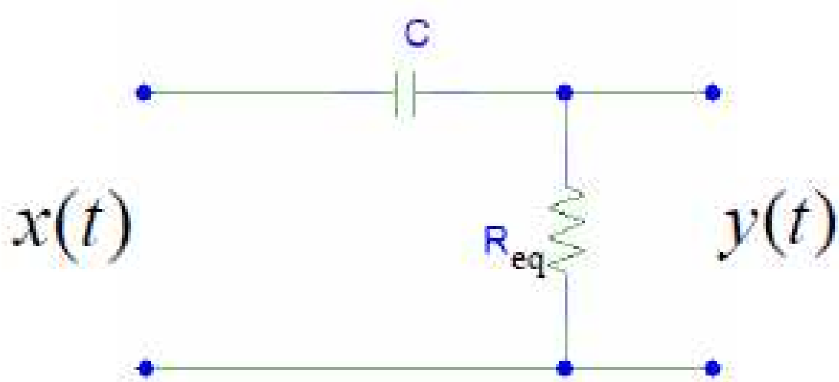
A rectangular pulse: $Ap_\tau(t)$



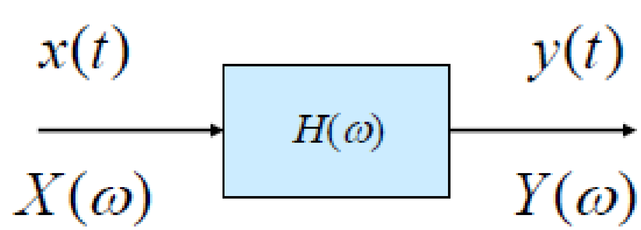
the interface circuitry consists of an "AC Coupled" transistor amplifier as shown below



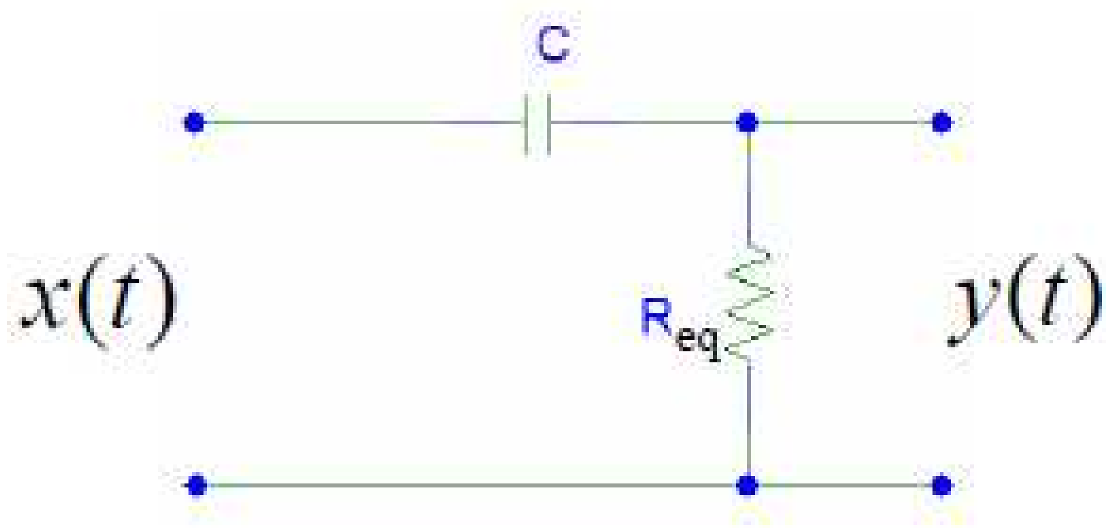
"Equivalent Circuit Model"



"Equivalent System Model"



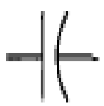
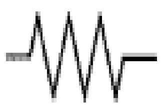
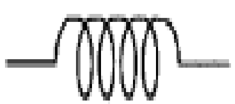
What is $H(\omega)$??



$$\Rightarrow \boxed{H(\omega) = \frac{j\omega R_{eq} C}{1 + j\omega R_{eq} C}}$$

Some Important rules that could be useful in solving RLC ccts can be shown in this table:

- **Table(4.1), Voltage-current, voltage-charge, and impedance relationships for capacitors, resistors, and inductors**

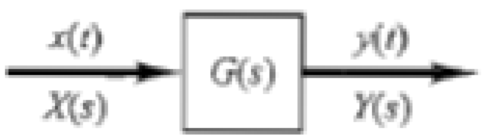
Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
 Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
 Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ – V (volts), $i(t)$ – A (amps), $q(t)$ – Q (coulombs), C – F (farads), R – Ω (ohms), G – Ω (mhos), L – H (henries).

Time Domain – Laplace and Frequency Domain Response of LTICT

Frequency response of LTI system

Consider the stable, linear, time-invariant system shown in Figure below. The input and output of the system, whose transfer function is *G(s) ,are* ***x(t) and y(t).*** If the input *x(t) is a sinusoidal signal, the output will be a sinusoidal signal* of the same frequency, but with possibly different **magnitude and phase angle**.
Let us assume that the input signal is given by x(t)= A cos wt , and the Laplace Transform of the Impulse response (Transfer Function of the system) G(s), Then Y(s)= G(s) X(s).
The frequency response can be calculated by replacing *s in the transfer function by jw. It will* also be shown that output will be given by:



$$G(j\omega) = Me^{j\phi} = M\,\underline{/\phi}$$

where M is the amplitude ratio of the output and input sinusoids and ϕ is the phase shift between the input sinusoid and the output sinusoid. In the frequency-response test, the input frequency *w is varied until the entire frequency range of interest is covered.*

Thus G(jw) which is a complex quantity is written by the following form:

$$G(j\omega) = \left|G(j\omega)\right| e^{j\phi}$$

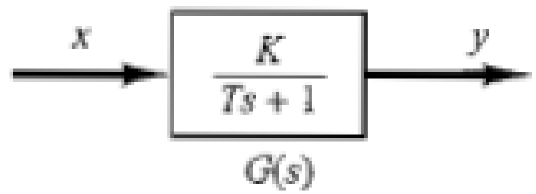
$$\phi = \underline{/G(j\omega)} = \tan^{-1}\left[\frac{\text{imaginary part of } G(j\omega)}{\text{real part of } G(j\omega)}\right]$$

$$\left|G(j\omega)\right| = \left|\frac{Y(j\omega)}{X(j\omega)}\right| = \text{amplitude ratio of the output sinuisoid to the input sinusoid}$$

$$\underline{/G(j\omega)} = \underline{/\frac{Y(j\omega)}{X(j\omega)}} = \text{phase shift of the output sinusoid with respect to the input sinusoid}$$

Example 1: Consider the system shown .The transfer function *G(s) is*

$$G(s) = \frac{K}{Ts + 1}$$



Substituting jw for *s in G(s) yields*

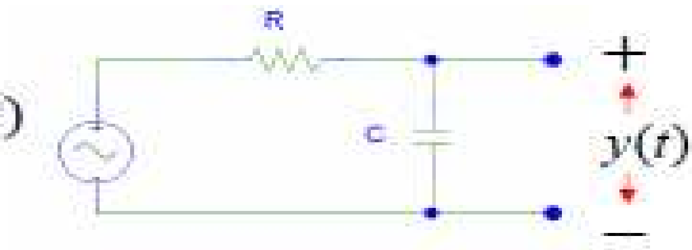
$$G(j\omega) = \frac{K}{jT\omega + 1} \qquad |G(j\omega)| = \frac{K}{\sqrt{1 + T^2\omega^2}} \qquad \phi = \angle G(j\omega) = -\tan^{-1} T\omega$$

Thus, for the input x(t) = X sin wt, the steady-state output y(t) can be obtained from Equation

$$\frac{XK}{\sqrt{1 + T^2\omega^2}} \sin\left(\omega t - \tan^{-1} T\omega\right)$$

Example 2: Consider the RC cct shown , find and plot its frequency response

$$x(t) = A\cos(\omega t + \theta)$$



$$x(t) = v_{in}(t)\,,\; y(t) = v_c(t)$$

$$v_{in}(t) = i(t)R + v_c(t)\;;\; Taking\;\; Laplace$$

$$I(s) = CsV_c(s)$$

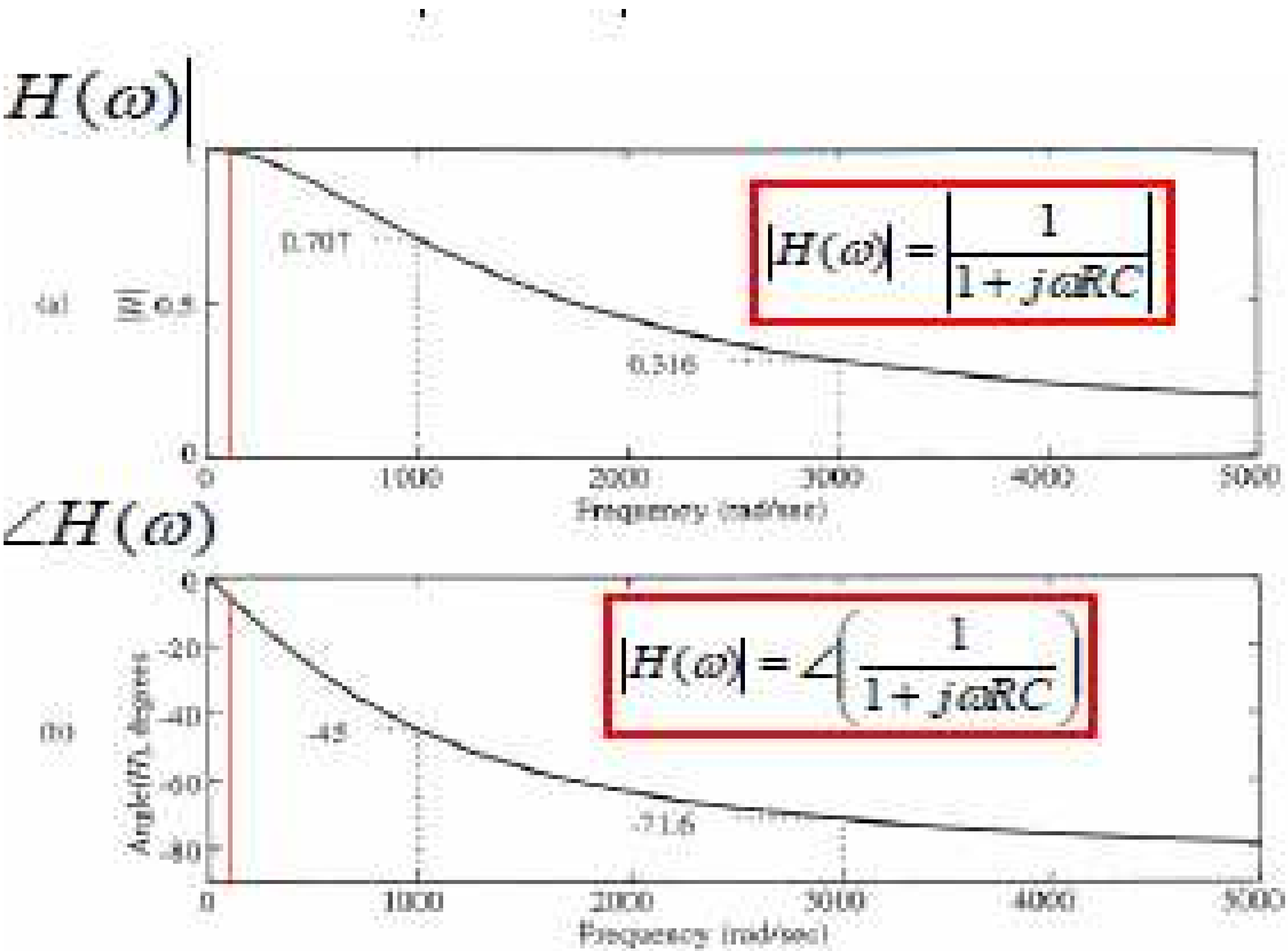
$$V_{in}(s) = RC\,sV_c(s) + V_c(s)$$

$$V_{in}(s) = (RCs + 1)V_c(s)$$

$$\frac{V_c(s)}{V_{in}(s)} = \frac{1}{RCs + 1} = \frac{Y(s)}{X(s)}$$

$$G(jw) = H(jw) = \frac{1}{RC\;jw + 1}$$

For 1/RC= 1000

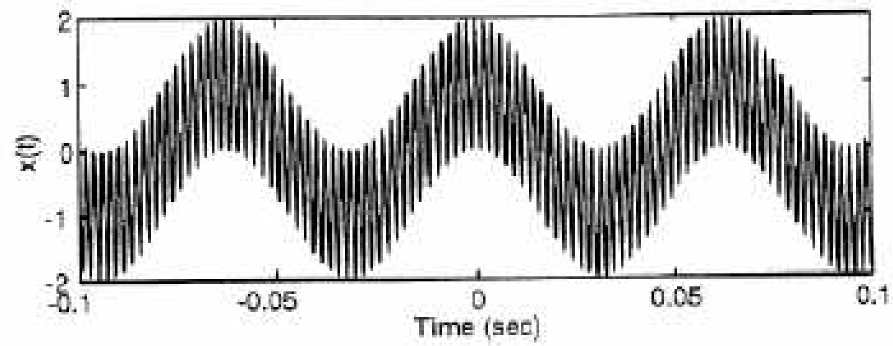


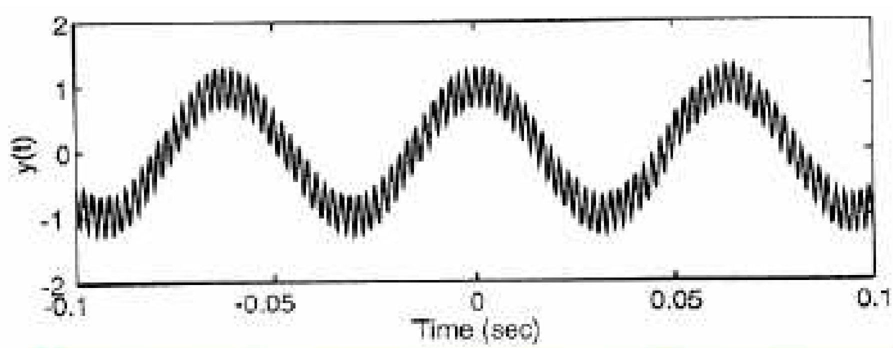
|H(ω)|
|H(ω)| = |1/(1+jωRC)|
0.707
0.316
(a)
|H| 0.5
0
0
1000
2000
3000
4000
5000
Frequency (rad/sec)
∠H(ω)
|H(ω)| = ∠(1/(1+jωRC))
0
-20
-40
-60
-80
-45
-71.6
(b)
Angle(H), degrees
0
1000
2000
3000
4000
5000
Frequency (rad/sec)

$$H(100) = 0.995e^{-j0.097}$$

$$H(3000) = 0.316e^{-j1.249}$$

$$x(t) = \cos(100t) + \cos(3000t)$$





$$y(t) = 0.995\cos(100t - 0.097) + 0.316\cos(3000t - 1.249)$$

Example 3 : Solving a First Order ODE

- Calculate the response of a CT LTI system with impulse response:

$$h(t) = e^{-bt}u(t) \qquad b > 0$$

- to the input signal:

$$x(t) = e^{-at}u(t) \qquad a > 0$$

- Taking Fourier transforms of both signals:

$$H(j\omega) = \frac{1}{b + j\omega}, \quad X(j\omega) = \frac{1}{a + j\omega}$$

- gives the overall frequency response:

$$Y(j\omega) = \frac{1}{(b + j\omega)(a + j\omega)}$$

- to convert this to the time domain, express as **partial fractions**:

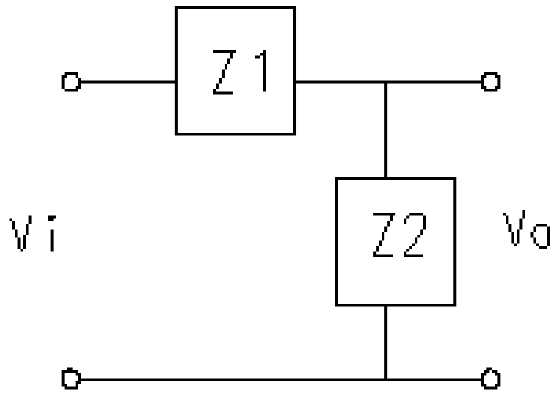
$$Y(j\omega) = \frac{1}{b - a}\left(\frac{1}{(a + j\omega)} - \frac{1}{(b + j\omega)}\right)$$

assume $b \neq a$

- Therefore, the CT system response is:

$$y(t) = \tfrac{1}{b-a}\left(e^{-at}u(t) - e^{-bt}u(t)\right)$$

First order Low Pass Filters:



$$\frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2}$$

Where Z= impedance =V/I

If Z_1 is a resistor and Z_2 is a capacitor then

$$\frac{V_o}{V_i} = \frac{\frac{1}{sC}}{\frac{1}{sC} + R} = \frac{1}{1 + sCR}$$

$$\left|\frac{V_o}{V_i}\right| = \left|\frac{1}{1 + sCR}\right| = \left|\frac{1}{1 + j\omega CR}\right| = \frac{1}{\sqrt{1^2 + (\omega CR)^2}}$$

If Z_1 is an inductor and Z_2 is a resistor another low pass structure

This is obviously a **low pass filter-LPF** (i.e., low frequency signals are passed and high frequency signals are blocked).

If w<<1/RC then wCR<<1 and the magnitude of the gain is approximately unity, and the output equals the input.

If w>>1/RC (wCR>>1) then the gain goes to zero, as does the output.

At w=1/RC, called the break frequency (or cutoff frequency, or 3dB frequency, or half-power frequency, or bandwidth), the magnitude of the gain is 1/sqrt(2).

In this case (and all first order RC circuits) high frequency is defined as w>>1/RC; the capacitor acts as a short circuit and all the voltage is across the resistance.

At low frequencies, w<<1/RC, the capacitor acts as an open circuit and there is no current (so the voltage across the resistor is near zero).

First order High Pass filter

If Z_1 is a capacitor and Z_2 is a resistor we can repeat the calculation:

$$\frac{V_o}{V_i} = \frac{R}{\frac{1}{sC} + R} = \frac{sCR}{1 + sCR}$$

$$\left|\frac{V_o}{V_i}\right| = \left|\frac{sCR}{1 + sCR}\right| = \left|\frac{j\omega CR}{1 + j\omega CR}\right| = \frac{\omega CR}{\sqrt{1^2 + (\omega CR)^2}}$$

At high frequencies, **w>>1/RC**, the capacitor acts as a short and the gain is 1 (the signal is passed).

At low frequencies, **w<<1/RC**, the capacitor is an open and the output is zero (the signal is blocked).

This is obviously a high pass structure and you can show that the break frequency is again 1/RC.

If Z_1 is a resistor and Z_2 is an inductor the resulting circuit is high pass with a break frequency of R/L.

Ideal filters

One of the most basic operations in any signal processing system is filtering. Filtering is the process by which the relative amplitudes of the frequency components in a signal are changed or perhaps some frequency components are suppressed. As we saw in the preceding section, for continuous-time LTI systems, the spectrum of the output is that of the input multiplied by the frequency response of the system. Therefore, an LTI system acts as a filter on the input signal. Here the word "filter" is used to denote a system that exhibits some sort of frequency-selective behavior.

A. Ideal Frequency-Selective Filters:

An ideal frequency-selective filter is one that exactly passes signals at one set of frequencies and completely rejects the rest. The band of frequencies passed by the filter is referred to as the pass band, and the band of frequencies rejected by the filter is called the stop band.
The most common types of ideal frequency-selective filters are the following

1. Ideal Low-Pass Filter:

An ideal low-pass filter (LPF) is specified by

$$|H(\omega)| = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$

which is shown in Fig. below (a). The frequency wc***, is called the cutoff frequency.***

2. Ideal High-Pass Filter:

An ideal high-pass filter (HPF) is specified by

$$|H(\omega)| = \begin{cases} 0 & |\omega| < \omega_c \\ 1 & |\omega| > \omega_c \end{cases}$$

which is shown in Fig. (b).

3. Ideal Bandpass Filter:

An ideal bandpass filter (BPF) is specified by

$$|H(\omega)| = \begin{cases} 1 & \omega_1 < |\omega| < \omega_2 \\ 0 & \text{otherwise} \end{cases}$$

which is shown in Fig. (c).

4. Ideal Bandstop Filter:

An ideal bandstop filter (BSF) is defined as

$$|H(\omega)| = \begin{cases} 0 & \omega_1 < |\omega| < \omega_2 \\ 1 & \text{otherwise} \end{cases}$$

which is shown in Fig. (d).

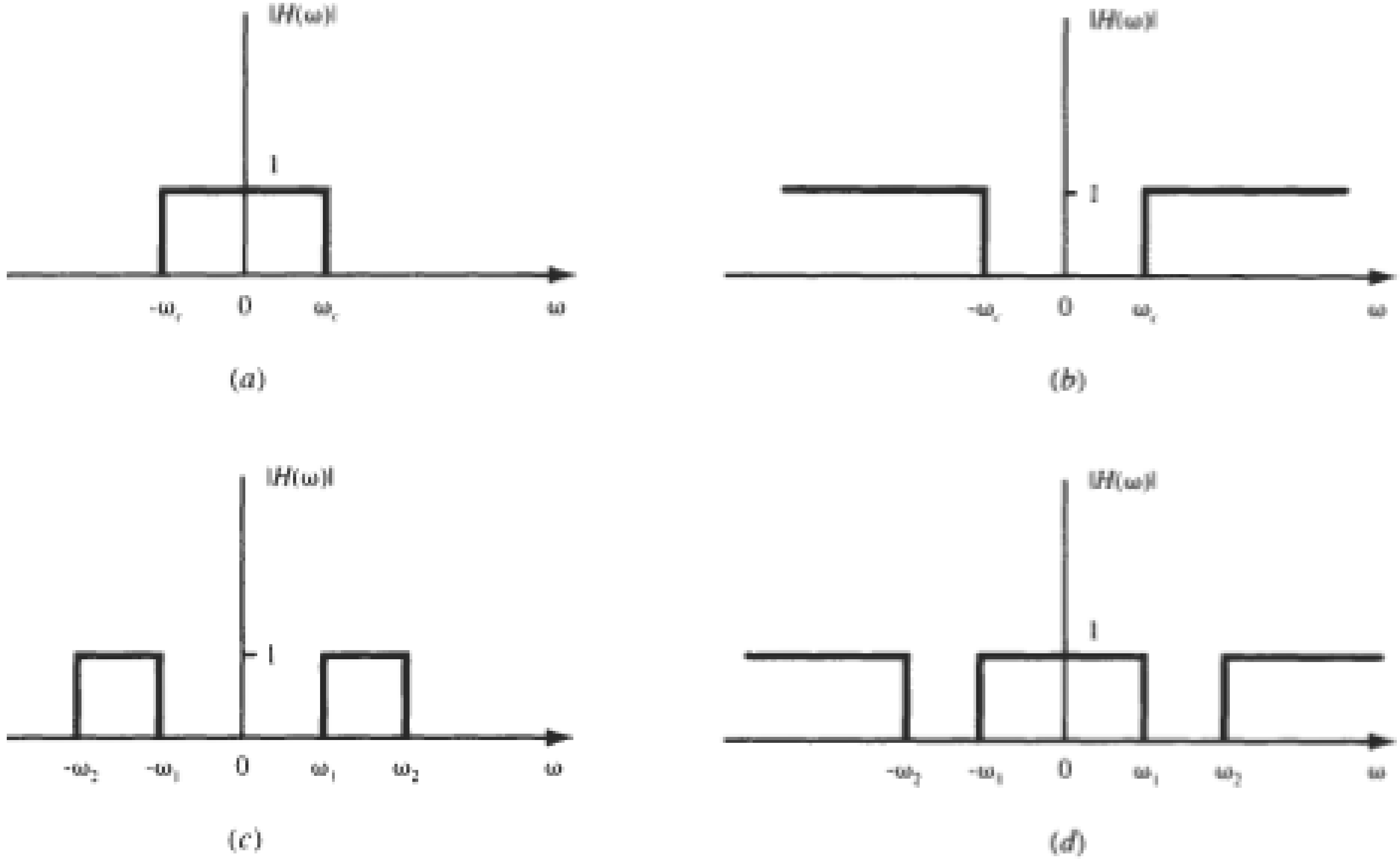


Figure (6.1), Magnitude responses of ideal frequency-selective filters

B. Non-Ideal Frequency-Selective Filters:

As an example of a simple continuous-time causal, frequency-selective filter, we consider the ***RC –LPF shown in figure(4.2)****. The output y(t) and the input x(t) are related* By:

$$RC\frac{dy(t)}{dt} + y(t) = x(t)$$

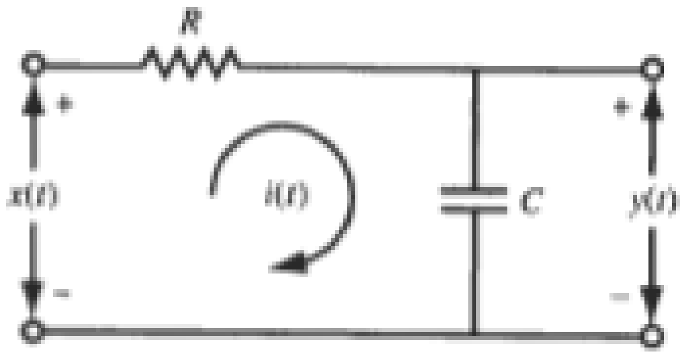
Taking the Fourier transforms of both sides of the above equation, the frequency response H(w) of the ***RC filter is given by***

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\omega/\omega_0}$$

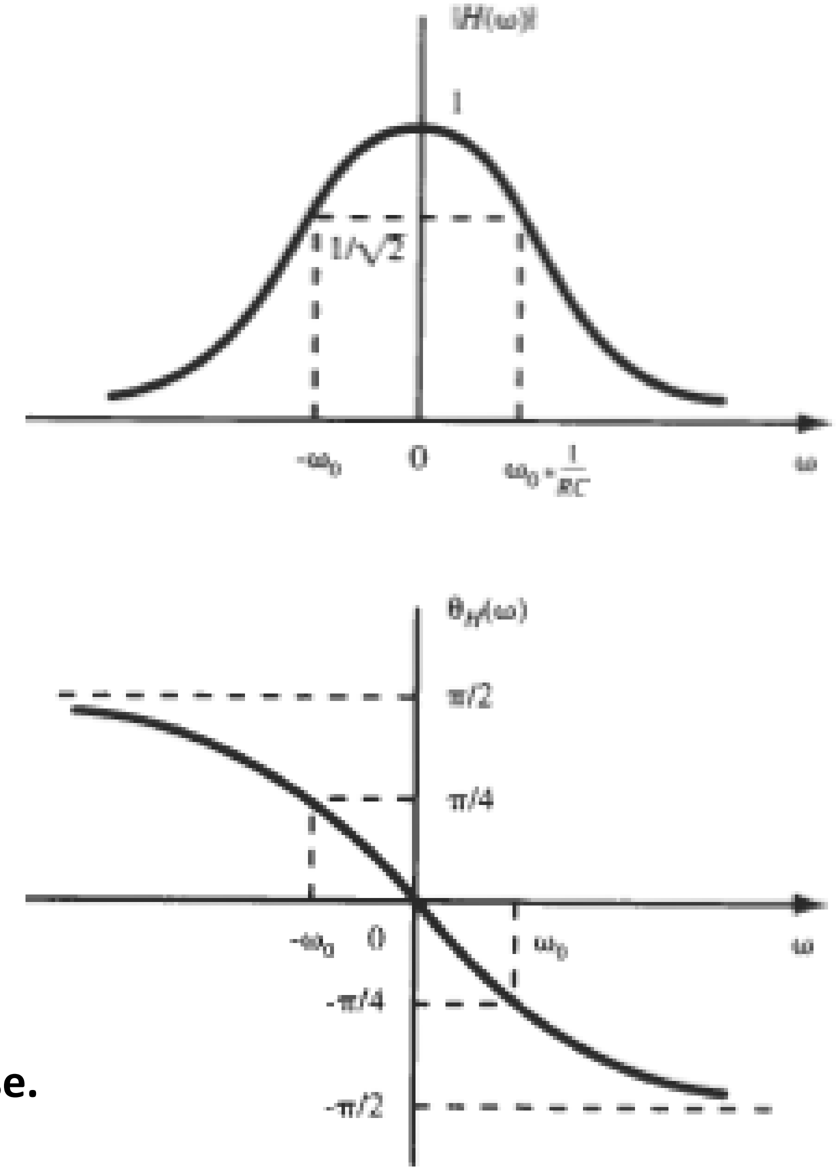
where ***wo = 1/RC. Thus, the amplitude***

$$|H(\omega)| = \frac{1}{|1 + j\omega/\omega_0|} = \frac{1}{\left[1 + (\omega/\omega_0)^2\right]^{1/2}}$$

$$\theta_H(\omega) = -\tan^{-1}\frac{\omega}{\omega_0}$$



(a)



(b)

Fig. 4.2 : RC filter and its frequency response.

Table (3): FT pairs for some functions and signals

CT signals	Time domain $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,dt$	Frequency domain $X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$
(1) Constant	1	$2\pi\delta(\omega)$
(2) Impulse function	$\delta(t)$	1
(3) Unit step function	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
(4) Causal decaying exponential function	$e^{-at}u(t)$	$\frac{1}{a+j\omega}$
(5) Two-sided decaying exponential function	$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$
(6) First-order time-rising causal decaying exponential function	$te^{-at}u(t)$	$\frac{1}{(a+j\omega)^2}$

(7) Nth-order time-rising causal decaying exponential function	$t^n e^{-at} u(t)$	$\dfrac{n!}{(a + j\omega)^{n+1}}$
(8) Sign function	$\mathrm{sgn}(t) = \begin{cases} 1 & t > 0 \\ -1 & t < 0 \end{cases}$	$\dfrac{2}{j\omega}$
(9) Complex exponential	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
(10) Periodic cosine function	$\cos(\omega_0 t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
(11) Periodic sine function	$\sin(\omega_0 t)$	$\dfrac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
(12) Causal cosine function	$\cos(\omega_0 t)u(t)$	$\dfrac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \dfrac{j\omega}{\omega_0^2 - \omega^2}$
(13) Causal sine function	$\sin(\omega_0 t)u(t)$	$\dfrac{\pi}{2j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \dfrac{\omega_0}{\omega_0^2 - \omega^2}$
(14) Causal decaying exponential cosine function	$e^{-at}\cos(\omega_0 t)u(t)$	$\dfrac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$
(15) Causal decaying exponential sine function	$e^{-at}\sin(\omega_0 t)u(t)$	$\dfrac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$