الجامعة السورية الخاصة
SYRIAN PRIVATE UNIVERSITY

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Faculty of Computer & Informatics and Communications Eng.

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Solve the following problems.

1. Consider a LTI system with a frequency response **[18 points]**

$$H(e^{j\omega}) = \frac{1}{2\left(1+\frac{1}{2}e^{-j\omega}\right)\left(1+3e^{-j\omega}\right)} \qquad (1)$$

a) Find a difference equation describing the system.
b) Calculate the DTFT of $x[n] = \delta[n-5] + 3\delta[n-6]$.
c) Find the DTFT $Y(e^{j\omega})$ of the output $y[n]$ when $x[n]$ is put through the system (1).
d) Find the output $y[n]$.

2. Using z-transform, compute **[14 M.]**

$y[n] = h[n] * x[n]$ for $h[n] = a^n u[n]$ and $x[n] = u[-n-1]$.

3. We want to design a Low Pass FIR Filter with the following characteristics: **[18 M.]**
Passband frequency 10kHz;
Stopband frequency 11kHz, with attenuation of 50dB;
Sampling frequency 44kHz.
Determine:

a) The specifications in the digital frequency domain;
b) The impulse response (h_d) of the ideal lowpass filter;
c) The causal impulse response $h[n]$ using one of the standard windows.

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With best success

Dr. Eng. Hassan Ahmad

Appendixes

Some common z-transform pairs			
#	**Sequence $x[n]$**	**z-Transform $X(z)$**	**ROC**
1.	$\delta[n]$	1	All z
2.	$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
3.	$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
4.	$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
5.	$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
6.	$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$z\rvert < \lvert a\rvert$
7.	$a^n u[n] - a^{n-1} u[n-1]$	$\frac{1-z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
8.	$(\cos\omega_0 n)u[n]$	$\frac{1-(\cos\omega_0)z^{-1}}{1-2(\cos\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
9.	$(\sin\omega_0 n)u[n]$	$\frac{(\sin\omega_0)z^{-1}}{1-2(\cos\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
10.	$(r^n\cos\omega_0 n)u[n]$	$\frac{1-(r\cos\omega_0)z^{-1}}{1-2(r\cos\omega_0)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$
11.	$(r^n\sin\omega_0 n)u[n]$	$\frac{(\sin\omega_0)z^{-1}}{1-2(r\cos\omega_0)z^{-1}+r^2z^{-2}}$	$\lvert z\rvert > r$

Fourier Transform Pairs for DTFT		
#	Sequence	Fourier Transform
1.	$\delta[n]$	1
2.	$\delta[n-n_0]$	$e^{-j\omega n_0}$
3.	1 $(-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega+2\pi k)$
4.	$a^n u[n] \quad (\lvert a\rvert<1)$	$\frac{1}{1-ae^{-j\omega}}$
5.	$(-1)^n a^n u[n] \quad (\lvert a\rvert<1)$, with $(-1)^n = \mathrm{e}^{-j\pi n}$	$\frac{1}{1+ae^{-j\omega}}$
6.	$u[n]$	$\frac{1}{1-ae^{-j\omega}} + \sum_{k=-\infty}^{\infty} 2\pi\delta(\omega+2\pi k)$
7.	$(n+1)a^n u[n] \quad (\lvert a\rvert<1)$	$\frac{1}{(1-ae^{-j\omega})^2}$
8.	$\frac{r^n \sin\omega_p(n+1)}{\sin\omega_p} u[n] \quad (\lvert r\rvert<1)$	$\frac{1}{1-2r\cos\omega_p e^{-j\omega}+r^2e^{-j2\omega}}$
9.	$\frac{\sin\omega_c n}{\pi n}$ = "sinc" function	$X(e^{j\omega}) = \begin{cases} 1, & \lvert\omega\rvert<\omega_c \\ 0, & \omega_c<\lvert\omega\rvert\le\pi \end{cases}$
10.	$x[n] = \begin{cases} 1, & 0\le n\le M \\ 0, & \text{otherwise} \end{cases}$	$\frac{\sin[\omega(M+1/2)]}{\sin(\omega/2)} e^{-j\omega M/2}$
11.	$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega-\omega_0+2\pi k)$
12.	$\cos(\omega_0 n+\phi)$	$\sum_{k=-\infty}^{\infty}\left[\pi e^{j\phi}\delta(\omega-\omega_0+2\pi k)+\pi e^{-j\phi}\delta(\omega+\omega_0+2\pi k)\right]$
13.	$\sin(\omega_0 n+\phi)$	$\sum_{k=-\infty}^{\infty}\left[-j\pi e^{j\phi}\delta(\omega-\omega_0+2\pi k)+j\pi e^{-j\phi}\delta(\omega+\omega_0+2\pi k)\right]$

Standard windows

Window	Function
Rectangular	$w[n]=\begin{cases}1, & 0\le n\le M\\ 0, & \text{otherwise}\end{cases}$
Bartlett (triangular)	$w[n]=\begin{cases}2n/M, & 0\le n\le M/2,\ M \text{ even}\\ 2-2n/M, & M/2<n\le M\\ 0, & \text{otherwise}\end{cases}$
Hanning	$w[n]=0.5-0.5\cos\left(\dfrac{2\pi n}{2M+1}\right),\quad -M\le n\le M$
Hamming	$w[n]=\begin{cases}0.54-0.46\cos(2\pi n/M), & 0\le n\le M\\ 0, & \text{otherwise}\end{cases}$
Blackman	$w[n]=\begin{cases}0.42-0.5\cos(2\pi n/M)+0.08\cos(2\pi n/M), & 0\le n\le M\\ 0, & \text{otherwise}\end{cases}$

Window name	Side lobe level (dB)	Approx. $\Delta\omega$	Exact $\Delta\omega$	$\delta_p \approx \delta_s$	A_p (dB)	A_s (dB)
Rectangular	−13	$4\pi/L$	$1.8\pi/L$	0.09	0.75	21
Bartlett	−25	$8\pi/L$	$6.1\pi/L$	0.05	0.45	26
Hann	−31	$8\pi/L$	$6.2\pi/L$	0.0063	0.055	44
Hamming	−41	$8\pi/L$	$6.6\pi/L$	0.0022	0.019	53
Blackman	−57	$12\pi/L$	$11\pi/L$	0.0002	0.0017	74